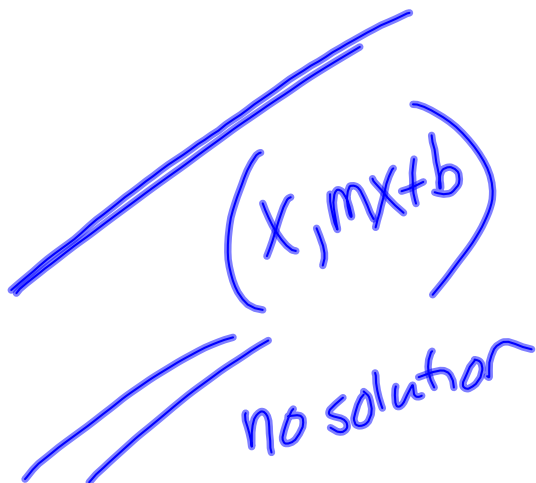
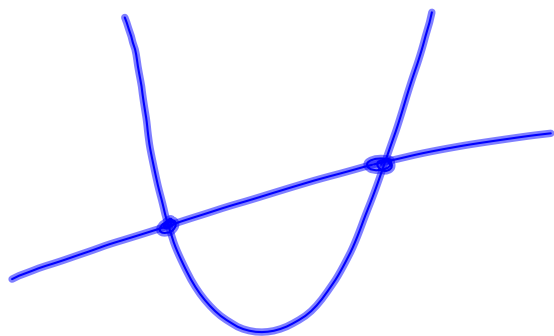


## 8.3 Systems of Equations & Matrices

substitution method

$$\begin{aligned}
 x + 3y &= 5 & x &= 5 - 3y = 5 - 3\left(\frac{7}{13}\right) \\
 2x - 7y &= 3 & &= \frac{65 - 21}{13} \\
 & & &= \frac{44}{13} \\
 2(5 - 3y) - 7y &= 3 & & \\
 10 - 6y - 7y &= 3 & & \\
 -13y &= -7 & & \\
 y &= \frac{7}{13} & &
 \end{aligned}$$

$$\left(\frac{44}{13}, \frac{7}{13}\right)$$

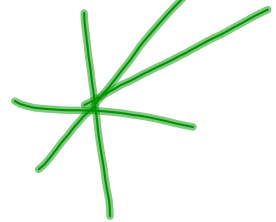
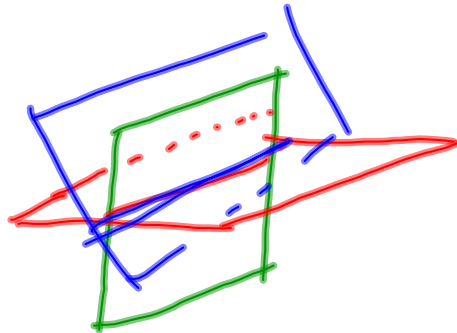
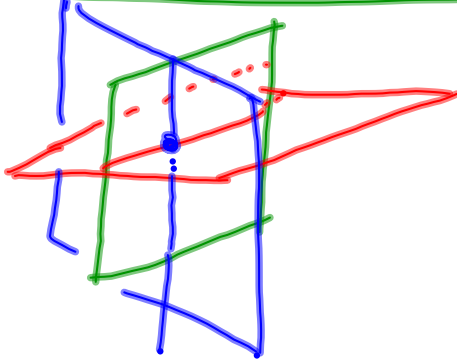


$$2x + 3y - z = 4$$

$$x - y + z = 2$$

$$x + y - 2z = 3$$

(x, y, z)



$$16. \begin{cases} 2x + y = 1 \\ 3x + 2y = -2 \end{cases} \Rightarrow \left[ \begin{array}{cc|c} 2 & 1 & 1 \\ 3 & 2 & -2 \end{array} \right]$$

augmented matrix

Gauss-Jordan Elimination

$$\left[ \begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array} \right] \text{ solution: } (a, b)$$

$$\left[ \begin{array}{cc|c} 2 & 1 & 1 \\ 3 & 2 & -2 \end{array} \right] \xrightarrow{R_1 \cdot \frac{1}{2}} \left[ \begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 3 & 2 & -2 \end{array} \right] \xrightarrow{R_2 - 3R_1} \left[ \begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{7}{2} \end{array} \right]$$

$2 \cdot \frac{1}{2} \quad -2 - 3(\frac{1}{2})$

$$\xrightarrow{R_2 \cdot 2} \left[ \begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -7 \end{array} \right] \xrightarrow{R_1 - \frac{1}{2} \cdot R_2} \left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -7 \end{array} \right]$$

$\frac{1}{2} + \frac{1}{2}(-7)$

solution:  $(4, -7)$

$$\begin{aligned} 28. \quad &x - y + 2z = 0 \\ &x - 2y + 3z = -1 \\ &2x - 2y + z = -3 \end{aligned}$$

or multiply  $R_1$  by fraction that gives you a 1

$$\begin{aligned} &\begin{matrix} -3 + (-2)(0) \\ -2 + (-2)(-1) \\ 1 + (-2)(2) \end{matrix} \begin{bmatrix} 1 & -1 & 2 & | & 0 \\ 1 & -2 & 3 & | & -1 \\ 2 & -2 & 1 & | & -3 \end{bmatrix} \xrightarrow{\text{swap rows}} \begin{bmatrix} 1 & -1 & 2 & | & 0 \\ 2 & -2 & 1 & | & -3 \\ 1 & -2 & 3 & | & -1 \end{bmatrix} \\ &\begin{matrix} R2 + (-1) \cdot R1 \\ R3 + (-2) \cdot R1 \end{matrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & | & 0 \\ 0 & -1 & 1 & | & -3 \\ 0 & 0 & -3 & | & -3 \end{bmatrix} \xrightarrow{R2 \cdot (-1)} \begin{bmatrix} 1 & -1 & 2 & | & 0 \\ 0 & 1 & -1 & | & 3 \\ 0 & 0 & -3 & | & -3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\begin{matrix} -1 + ( ) 1 \\ R1 + (1) \cdot R2 \\ R3 + ( ) \cdot R2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & | & 3 \\ 0 & 1 & -1 & | & 3 \\ 0 & 0 & -3 & | & -3 \end{bmatrix} \xrightarrow{R3 \cdot (-\frac{1}{3})} \begin{bmatrix} 1 & 0 & 1 & | & 3 \\ 0 & 1 & -1 & | & 3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\begin{matrix} R1 + (-1) \cdot R3 \\ R2 + (1) \cdot R3 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & 4 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \quad \text{solution: } (0, 2, 1) \end{aligned}$$

$$\begin{aligned} 32. \quad &\begin{cases} 2x - 3y + 2z = 2 \\ x + 4y - z = 9 \\ -3x + y - 5z = 5 \end{cases} \quad \begin{bmatrix} 2 & -3 & 2 & | & 2 \\ 1 & 4 & -1 & | & 9 \\ -3 & 1 & -5 & | & 5 \end{bmatrix} \xrightarrow{\begin{matrix} R_1 \leftrightarrow R_2 \\ \text{OR} \\ R_i ( ) \end{matrix}} \begin{bmatrix} 1 & 4 & -1 & | & 9 \\ 2 & -3 & 2 & | & 2 \\ -3 & 1 & -5 & | & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\begin{matrix} R2 + (-2) \cdot R1 \\ R3 + (3) \cdot R1 \end{matrix} \rightarrow \begin{bmatrix} 1 & 4 & -1 & | & 9 \\ 0 & -11 & 4 & | & -16 \\ 0 & 13 & -8 & | & 32 \end{bmatrix} \xrightarrow{R2 \cdot (-\frac{1}{11})} \begin{bmatrix} 1 & 4 & -1 & | & 9 \\ 0 & 1 & -\frac{4}{11} & | & \frac{16}{11} \\ 0 & 13 & -8 & | & 32 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\begin{matrix} R1 + (-4) \cdot R2 \\ R3 + (-13) \cdot R2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{5}{11} & | & \frac{35}{11} \\ 0 & 1 & -\frac{4}{11} & | & \frac{16}{11} \\ 0 & 0 & -\frac{36}{11} & | & \frac{11}{11} \end{bmatrix} \xrightarrow{R3 \cdot (-\frac{11}{36})} \begin{bmatrix} 1 & 0 & \frac{5}{11} & | & \frac{35}{11} \\ 0 & 1 & -\frac{4}{11} & | & \frac{16}{11} \\ 0 & 0 & 1 & | & -\frac{1}{4} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\begin{matrix} R1 + (-\frac{5}{11}) \cdot R3 \\ R2 + (\frac{4}{11}) \cdot R3 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & \frac{35}{11} + \frac{5}{44} \\ 0 & 1 & 0 & | & \frac{16}{11} - \frac{1}{11} \\ 0 & 0 & 1 & | & -\frac{1}{4} \end{bmatrix} \quad \text{solution: } (5, 0, -\frac{1}{4}) \end{aligned}$$

8.3  

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#27, 31