8.3 Systems of Equations & Matrices

Substitution method:

$x \quad + 3y = 5$

$2x - \ 7y = 3$

$x = -3y + 5$

$2(-3y+5) - 7y = 3$

$-6y + 10 - 7y = 3$

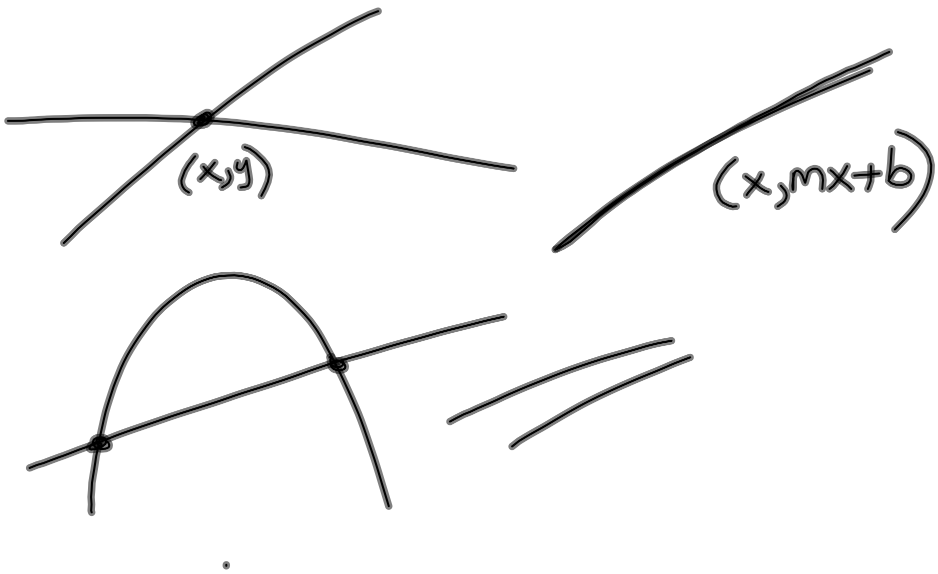
$-13y = -7$

$y = \frac{7}{13}$

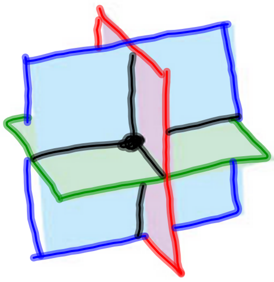
$x = -3\left(\frac{7}{13}\right) + 5$

$= \frac{-21}{13} + \frac{65}{13} = \frac{44}{13}$

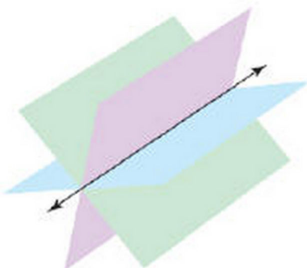
$\boxed{\left(\frac{44}{13}, \frac{7}{13}\right)}$



$$\begin{cases} 2x+4y-z=7 \\ x-y+5z=2 \\ 3x+2y+z=-3 \end{cases}$$



(a) One solution (a point)



(b) Infinite number of solutions (a line)

(c) Infinite number of solutions (a plane)

(d) No solution

(e) No solution

16. $\begin{cases} 2x+y=1 \\ 3x+2y=-2 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \\ 3 & 2 & -2 \end{array}\right]$

augmented matrix

Gauss-Joran Elimination

$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$ Solution: (a,b)

$R_1 \cdot \frac{1}{2}$

$\left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 3 & 2 & -2 \end{array}\right] \xrightarrow{R_2+(-3)R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{7}{2} \end{array}\right]$

$2+(-3)\cdot\frac{1}{2}=$

$-2+(-3)\cdot\frac{1}{2}=$

$\xrightarrow{R_2\cdot 2} \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -7 \end{array}\right]$

$R_1+(-\frac{1}{2})R_2$

$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -7 \end{array}\right]$

$\frac{1}{2}+(-\frac{1}{2})(-7)$

$\boxed{(4,-7)}$

28. $x-y+2z=0$
$x-2y+3z=-1$
$2x-2y+z=-3$

$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & -2 & 3 & -1 \\ 2 & -2 & 1 & -3 \end{array}\right] \xrightarrow{\textit{swap rows}} \cancel{\left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]}$

OR mult. $R_1\cdot(\text{const})$

$-2+(-1)(-1)=-2+1=-1$

$3+(-1)(2)=3-2=1$

$-1+(-1)(0)=-1$

$R2+(-1)\cdot R1$

$R3+(-2)\cdot R1$

$\longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right] \xrightarrow{R2\cdot(-1)} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right]$

$-2+(-2)(-1)=-2+2=0$

$1+(-2)(2)=-3$

$-3+(-2)(0)=-3$

$R1+(1)\cdot R2$

~~$R3+(\ )\cdot R2$~~

$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right] \xrightarrow{R3\cdot(-\frac{1}{3})} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$1+(-1)(1)=0$

$1+1(1)=2$

$R1+(-1)\cdot R3$

$R2+(1)\cdot R3$

$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\boxed{\textit{solution: } (0, 2, 1)}$

32.

$$\begin{cases} 2x-3y+2z=2 \\ x+4y-z=9 \\ -3x+y-5z=5 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 2 \\ 1 & 4 & -1 & 9 \\ -3 & 1 & -5 & 5 \end{array}\right] \xrightarrow[R_1 \,\&\, R_2]{\textit{swap rows}} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 9 \\ 2 & -3 & 2 & 2 \\ -3 & 1 & -5 & 5 \end{array}\right]$$

$$\xrightarrow[R3+(3)\cdot R1]{R2+(-2)\cdot R1} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 9 \\ 0 & -11 & 4 & -16 \\ 0 & 13 & -8 & 32 \end{array}\right] \xrightarrow{R2\cdot\left(-\frac{1}{11}\right)} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 9 \\ 0 & 1 & -\frac{4}{11} & \frac{16}{11} \\ 0 & 13 & -8 & 32 \end{array}\right]$$

$-3+(-2)(4)$

$2+(-2)(-1)$

$2+(-2)(9)$

$1+3(4)$

$-5+3(-1)$

$5+3(9)$

$$\xrightarrow[R3+(-13)\cdot R2]{R1+(-4)\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{11} & \frac{35}{11} \\ 0 & 1 & -\frac{4}{11} & \frac{16}{11} \\ 0 & 0 & -\frac{36}{11} & \frac{144}{11} \end{array}\right] \xrightarrow{R3\cdot\left(-\frac{11}{36}\right)} \left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{11} & \frac{35}{11} \\ 0 & 1 & -\frac{4}{11} & \frac{16}{11} \\ 0 & 0 & 1 & -4 \end{array}\right]$$

$-1+(-4)\left(-\frac{4}{11}\right)$

$-\frac{11}{11}+\frac{16}{11}=$

$9+(-4)\left(\frac{16}{11}\right)$

$-8+(-13)\left(-\frac{4}{11}\right)$

$\frac{-8(11)+13(4)}{11}=\frac{-36}{11}$

$32+(-13)\left(\frac{16}{11}\right)=$

$\frac{32(11)-13(16)}{11}=$

$\frac{144}{11}\cdot\frac{-11}{36}$

$$\xrightarrow[R2\,+\,\left(\frac{4}{11}\right)\cdot R3]{R1\,+\,\left(-\frac{5}{11}\right)\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 \end{array}\right] \qquad \boxed{\textit{solution: } (5,\ 0,\ -4)}$$

$\frac{35}{11}+\left(\frac{-5}{11}\right)(-4)$

$\frac{16}{11}+\frac{4}{11}(-4)$

8.3

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