$$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases} \qquad 30. \begin{cases} 3x+2y+2z=3 \\ x+2y-z=5 \\ 2x-4y+z=0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 2 & 3 \\ 1 & 2 & -1 & 5 \\ 2 & -4 & 1 & 0 \end{array}\right] \xrightarrow[R_1 \& R_2]{\textit{swap rows}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 3 & 2 & 2 & 3 \\ 2 & -4 & 1 & 0 \end{array}\right]$$

$2+(-3)2$ $\quad -4+(-2)(2)$

$2+(-3)(-1)$ $\quad 1+(-2)(-1)$

$3+(-3)(5)$ $\quad 0+(-2)(5)$

$$\xrightarrow[R3+(-2)\cdot R1]{R2+(-3)\cdot R1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -4 & 5 & -12 \\ 0 & -8 & 3 & -10 \end{array}\right] \xrightarrow{R2\cdot(-\frac{1}{4})} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & -5/4 & 3 \\ 0 & -8 & 3 & -10 \end{array}\right]$$

$-1+(-2)(-5/4)$ $\quad 3+8(-5/4)$

$5+(-2)(3)$ $\quad -10+8(3)$

$$\xrightarrow[R3+(8)\cdot R2]{R1+(-2)\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & 3/2 & -1 \\ 0 & 1 & -5/4 & 3 \\ 0 & 0 & -7 & 14 \end{array}\right] \xrightarrow{R3\cdot(-\frac{1}{7})} \left[\begin{array}{ccc|c} 1 & 0 & 3/2 & -1 \\ 0 & 1 & -5/4 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$-1+(-3/2)(-2)$

$=2$

$3+(5/4)(-2)$ $\quad \frac{6}{2}-\frac{5}{2}=\frac{1}{2}$

$$\xrightarrow[R2+(5/4)\cdot R3]{R1+(-3/2)\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -2 \end{array}\right] \qquad \boxed{\textit{solution: } (2, 1/2, -2)}$$

$$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases} \qquad 36. \begin{cases} m+n+t=9 \\ m-n-t=-15 \\ 3m+n+t=2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -1 & -1 & -15 \\ 3 & 1 & 1 & 2 \end{array}\right] \xrightarrow{\textit{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$$

$-1+(-1)(1)$ $\quad 1+(-3)(1)$

$-1+(-1)(1)$ $\quad 1+(-3)(1)$

$-15+(-1)(9)$ $\quad 2+(-3)(9)$

$$\xrightarrow[R3+(-3)\cdot R1]{R2+(-1)\cdot R1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & -2 & -2 & -25 \end{array}\right] \xrightarrow[R_3+(-1)R_2]{\cancel{R2\cdot(\ )}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$0x+0y+0z=-1$

$0=-1$

$$\xrightarrow[R3+(\ )\cdot R2]{R1+(\ )\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right] \xrightarrow{R3\cdot(\ )} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right]$$

contradictions occur when a row has all zeros on the left (all zero variable coefficients) & nonzero constant on the right ⇒

contradiction

no solution

$$\xrightarrow[R2+(\ )\cdot R3]{R1+(\ )\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right] \qquad \boxed{\textit{solution: } (\ \ ,\ \ ,\ \ )}$$

$$\begin{cases} ax + by + cz = d \\ ax + by + cz = d \\ ax + by + cz = d \end{cases}$$

34. $x+y-3z=4$

$4x+5y+z=1$

$2x+3y+7z=-7$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 4 & 5 & 1 & 1 \\ 2 & 3 & 7 & -7 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$$

$5+(-4)(1)$ $\quad 3+(-2)(1)$

$1+(-4)(-3)$ $\quad 7+(-2)(-3)$

$1+(-4)(4)$ $\quad -7+(-2)(4)$

$$\xrightarrow[R3+(-2)\cdot R1]{R2+(-4)\cdot R1} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 1 & 13 & -15 \end{array}\right] \xrightarrow[R_3+(-R_2)]{\cancel{R2\cdot(\ \ )}} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0x+0y+0z=0$

$0=0$

Row of all zeros $\Rightarrow$ infinitely many solutions we need to determine their form

$$\xrightarrow[R3+(\ \ )\cdot R2]{R1+(\ \ )\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right] \xrightarrow{R3\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right]$$

$x+y-3z=4$

$y+13z=-15$

$y=-13z-15$

$x=-y+4+3z$

$=-(-13z-15)+3z+4$

$=13z+15+3z+4$

$x=16z+19$

solution: $(16z+19, -13z-15, z)$

$$\begin{cases} ax + by + cz = d \\ ax + by + cz = d \\ ax + by + cz = d \end{cases}$$

35. $p+q+r=1$

$p+2q+3r=4$

$4p+5q+6r=7$

$p+q+r=1$

$q+2r=3$

$\to q=3-2r$

$\to p=-q-r+1$

$=-(3-2r)-r+1=-3+2r-r+1=-2+r$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$$

$(-2+r, 3-2r, r)$

$$\xrightarrow[R3+(\ \ )\cdot R1]{R2+(\ \ )\cdot R1} \left[\begin{array}{ccc|c} 1 & & & \\ 0 & & & \\ 0 & & & \end{array}\right] \xrightarrow{R2\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & & & \\ 0 & 1 & & \\ 0 & & & \end{array}\right]$$

$$\xrightarrow[R3+(\ \ )\cdot R2]{R1+(\ \ )\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right] \xrightarrow{R3\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right]$$

$$\xrightarrow[R2+(\ \ )\cdot R3]{R1+(\ \ )\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right]$$

$\boxed{\textit{solution: } (\ \ ,\ \ ,\ \ )}$

$$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases}$$

37. $a+b-c=7$
$a-b+c=5$
$3a+b-c=-1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 1 & 5 \\ 3 & 1 & -1 & -1 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$$

$-1+(-1)(1)=-2$
$1+(-1)(-1)=2$
$5+(-1)(7)$

$1+(-3)(1)$
$-1+(-3)(-1)$
$-1+(-3)(7)$

$$\xrightarrow[R3+(-3)\cdot R1]{R2+(-1)\cdot R1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & -2 & 2 & -22 \end{array}\right] \xrightarrow{R2\cdot(\quad)} \left[\begin{array}{ccc|c} 1 & & & \\ 0 & 1 & & \\ 0 & & & \end{array}\right]$$

$R_3+(-R_2)$

$0 \quad 0 \quad -20$

$0=-20$

no solution!

$$\xrightarrow[R3+(\quad)\cdot R2]{R1+(\quad)\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right] \xrightarrow{R3\cdot(\quad)} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right]$$

$$\xrightarrow[R2+(\quad)\cdot R3]{R1+(\quad)\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right]$$

$\boxed{\textit{solution: } (\quad,\quad,\quad)}$

HW
8.2 (but solve using a matrix)
#3, 7, 9

3. $x-y+2z=-3$
$x+2y+3z=4$
$2x-y-z=-3$

ans: $(-3,2,1)$

7. $x+2y-z=-8$
$2x-y+z=4$
$8x+y+z=2$

ans: no solution

9. $2x+y-3z=1$
$x-4y+z=6$
$4x-7y-z=13$

ans: $\left(\frac{11y+19}{5}, y, \frac{9y+11}{5}\right)$