$$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases}$$

30. $$\begin{cases} 3x+2y+2z=3 \\ x+2y-z=5 \\ 2x-4y+z=0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 2 & 3 \\ 1 & 2 & -1 & 5 \\ 2 & -4 & 1 & 0 \end{array}\right] \xrightarrow[R_1 \& R_2]{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 3 & 2 & 2 & 3 \\ 2 & -4 & 1 & 0 \end{array}\right]$$

$2+(-3)(2)$ $\quad -4+(-2)(2)$

$2+(-3)(-1)$ $\quad 1+(-2)(-1)$

$3+(-3)(5)$ $\quad 0+(-2)(5)$

$$\xrightarrow[R3+(-2)\cdot R1]{R2+(-3)\cdot R1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -4 & 5 & -12 \\ 0 & -8 & 3 & -10 \end{array}\right] \xrightarrow{R2\cdot(-\frac{1}{4})} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & -5/4 & 3 \\ 0 & -8 & 3 & -10 \end{array}\right]$$

$-1+(-2)(\frac{-5}{4})$

$-\frac{4}{4}+\frac{10}{4}=\frac{6}{4}=\frac{3}{2}$

$5+(-2)(3)$

$3+8(\frac{-5}{4})=3-10$

$-10+8(3)=$

$$\xrightarrow[R3+(8)\cdot R2]{R1+(-2)\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & 3/2 & -1 \\ 0 & 1 & -5/4 & 3 \\ 0 & 0 & -7 & 14 \end{array}\right] \xrightarrow{R3\cdot(-\frac{1}{7})} \left[\begin{array}{ccc|c} 1 & 0 & 3/2 & -1 \\ 0 & 1 & -5/4 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$-1+(-\frac{3}{2})(-2)$

$3+\frac{5}{4}(-2)=\frac{6}{2}-\frac{5}{2}$

$$\xrightarrow[R2+(\frac{5}{4})\cdot R3]{R1+(-\frac{3}{2})\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\boxed{\textit{solution: } (2, \tfrac{1}{2}, -2)}$

$$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases}$$

36. $m+n+t=9$

$m-n-t=-15$

$3m+n+t=2$

$-1+(-1)1$

$-1+(-1)1$

$-15+(-1)9$

$1+(-3)1$

$1+(-3)1$

$2+(-3)9$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -1 & -1 & -15 \\ 3 & 1 & 1 & 2 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$$

$$\xrightarrow[R3+(-3)R1]{R2+(-1)R1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & -2 & -2 & -25 \end{array}\right] \xrightarrow[R3+(-R2)]{R2\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$-24+(+25)$

contradiction occurs:

zeros on the left (variable coefficients) & non-zero constant on right

$$\xrightarrow[R3+(\ \ )\cdot R2]{R1+(\ \ )\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right] \xrightarrow{R3\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right]$$

$0x+0y+0z=1$

$0=1$

contradiction

$\boxed{\text{no solution}}$

$$\xrightarrow[R2+(\ \ )\cdot R3]{R1+(\ \ )\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right]$$

$\boxed{\textit{solution: } (\quad,\quad,\quad)}$

$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases}$

34. $x+y-3z=4$
$4x+5y+z=1$
$2x+3y+7z=-7$

$5+(-4)1$
$1+(-4)(-3)$
$1+(-4)(4)$

$3+(-2)1$
$7+(-2)(-3)$
$-7+(-2)4$

$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 4 & 5 & 1 & 1 \\ 2 & 3 & 7 & -7 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$

$\xrightarrow[R3+(-2)R1]{R2+(-4)R1} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 1 & 13 & -15 \end{array}\right] \xrightarrow{R_3+(-R_2)} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$0x+0y+0z=0$
$0=0$

Row of all zeros
$\Rightarrow$ infinitely many solutions
we need to determine their form

$\xrightarrow[R3+(\ )\cdot R2]{R1+(\ )\cdot R2} \begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & \end{bmatrix} \xrightarrow{R3\cdot(\ )} \begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$

$x+y-3z=4 \rightarrow x=-y+3z+4$

$y+13z=-15$

$=-(-13z-15)+3z+4$
$=13z+15+3z+4$
$x=16z+19$

$y=-13z-15$

$\xrightarrow[R2+(\ )\cdot R3]{R1+(\ )\cdot R3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ solution: (, ,)

$\boxed{(16z+19, -13z-15, z)}$

$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases}$

35. $p+q+r=1$
$p+2q+3r=4$
$4p+5q+6r=7$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$

$p+q+r=1$
$q+2r=3$
$q=3-2r$

$p+(3-2r)+r=1$
$p=1-r-3+2r$
$=-2+r$

$\xrightarrow[R3+(\ )\cdot R1]{R2+(\ )\cdot R1} \begin{bmatrix} 1 & & \\ 0 & & \\ 0 & & \end{bmatrix} \xrightarrow{R2\cdot(\ )} \begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & & \end{bmatrix}$

$(f(q), q, g(q))$
$(p, f(p), g(p))$

$\boxed{(r-2, 3-2r, r)}$

$\xrightarrow[R3+(\ )\cdot R2]{R1+(\ )\cdot R2} \begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & \end{bmatrix} \xrightarrow{R3\cdot(\ )} \begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$

$\xrightarrow[R2+(\ )\cdot R3]{R1+(\ )\cdot R3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ solution: (, ,)

$$\begin{cases} ax+by+cz=d \\ ax+by+cz=d \\ ax+by+cz=d \end{cases}$$

37. $a+b-c=7$
$a-b+c=5$
$3a+b-c=-1$

$-1+(-1)1$
$1+(-1)(-1)$
$5+(-1)(7)$

$1+(-3)1$
$-1+(-3)(-1)$
$-1+(-3)(7)$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 1 & 5 \\ 3 & 1 & -1 & -1 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right]$$

$$\xrightarrow[R3+(-3)R1]{R2+(-1)R1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & -2 & 2 & -22 \end{array}\right] \xrightarrow[R_3-R2]{R2\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & 0 & -20 \end{array}\right]$$

$-22-(-2)$

$0=-20$

$$\xrightarrow[R3+(\ \ )\cdot R2]{R1+(\ \ )\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right] \xrightarrow{R3\cdot(\ \ )} \left[\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right]$$

no solution!

$$\xrightarrow[R2+(\ \ )\cdot R3]{R1+(\ \ )\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right]$$

$\boxed{solution:\ (\quad,\quad,\quad)}$

Hw
8.2 (but solve using a matrix)
#3, 7, 9

3. $x-y+2z=-3$
$x+2y+3z=4$
$2x-y-z=-3$

ans: $(-3,2,1)$

7. $x+2y-z=-8$
$2x-y+z=4$
$8x+y+z=2$

ans: no solution

9. $2x+y-3z=1$
$x-4y+z=6$
$4x-7y-z=13$

ans: $\left(\frac{11y+19}{5}, y, \frac{9y+11}{5}\right)$