6. $\log_5 \frac{1}{125} = \boxed{-3}$ $\quad 5^x = \frac{1}{125} = \frac{1}{5^3} = 5^{-3}$

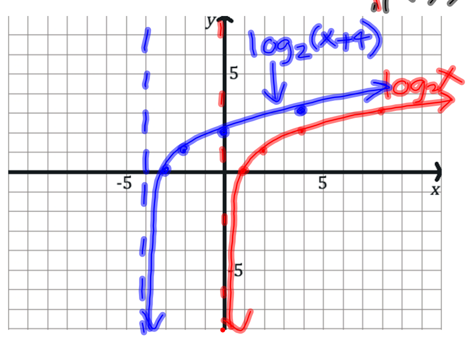
7. $\log_{27} 3 = \boxed{\frac{1}{3}}$ $\quad 27^x = 3 \quad \sqrt[3]{27} = 3$

8. $3^{2\log_3 2} = 3^{\log_3 2^2} = 3^{\log_3 4} = \boxed{4}$

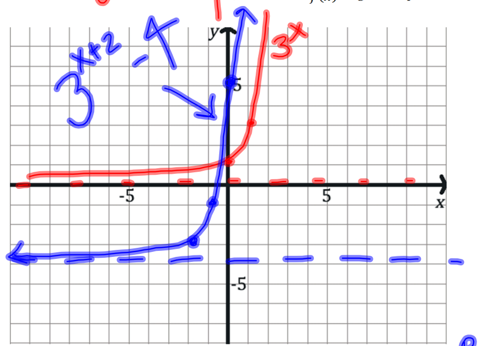
9. $\ln \frac{e^2}{\sqrt[3]{e}} = \ln e^2 - \ln e^{1/3} = 2 - \frac{1}{3} = \frac{6}{3} - \frac{1}{3} = \boxed{\frac{5}{3}}$

10. $\log_4 8 = \frac{\log_2 8}{\log_2 4} = \boxed{\frac{3}{2}}$

$f(x) = log_2(x+4)$ left 4 $\log_a x$ $(a,1)$ $(0,1)$




$a^x$ $(1,a)$ $(0,1)$ left 2, down 4 $f(x) = 3^{x+2} - 4$




14. Express as a single logarithm.

$4 \ln x - 2[\ln 3y + 3 \ln 2x] = 4\ln x - 2[\ln 3y + \ln (2x)^3]$ $\quad p \log_a x = \log_a x^p$

$= \ln x^4 - 2 \ln (3y \cdot 8x^3)$

$= \ln x^4 - \ln (24x^3 y)^2$

$= \ln\left(\frac{x^4}{24^2 x^6 y^2}\right) = \ln\left(\frac{1}{576 x^2 y^2}\right)$

16. Solve for x.

$$3^{5x+2} = \frac{1}{27}$$

$$3^{5x+2} = 3^{-3}$$

$$5x+2 = -3$$

$$5x = -5$$

$$\boxed{x = -1}$$

17. Solve for x.

$$\log_3 x + \log_3 (x-8) = 2$$

$$\log_3 [x(x-8)] = 2$$

$$3^2 = x(x-8)$$

$$9 = x^2 - 8x$$

$$x^2 - 8x - 9 = 0$$

$$(x-9)(x+1) = 0$$

$$\boxed{x = 9}$$

$$\cancel{x = -1}$$

18. A certain element has a half-life of 30 years and we start with a 4000 gram sample of the element.

$$P(30) = \frac{1}{2}P_0$$

$$P(t) = P_0 e^{kt}$$

$$\frac{1}{2}P_0 = P_0 e^{kt}$$

$$2000 = 4000e^{k30}$$

$$\frac{1}{2} = e^{k30}$$

$$\ln\frac{1}{2} = k \cdot 30$$

$$k = \frac{\ln\frac{1}{2}}{30}$$

$$\boxed{P(t) = 4000e^{\frac{\ln\frac{1}{2}}{30}t}}$$

c. Determine the number of years $t$ it will take for there to be only 100 grams of the element left. in terms of the natural log.

what is t, when $P(t) = 100$?

$$100 = 4000e^{kt}$$

$$\frac{100}{4000} = e^{kt}$$

$$\frac{1}{40} = e^{kt}$$

$$\ln\frac{1}{40} = kt$$

$$t = \frac{\ln\frac{1}{40}}{k} = \boxed{\frac{\ln\frac{1}{40}}{\frac{1}{30}\ln\frac{1}{2}}}$$

$$= \frac{\ln(40)^{-1}}{\frac{1}{30}\cdot\ln(2)^{-1}} = \frac{30(-\ln 40)}{-\ln 2} = \boxed{\frac{30\ln 40}{\ln 2}}$$

solution to # 3 as written on board:
$\left(-3/8, 19/8, -1/8\right)$

# 10.1 Sequences and Series

An infinite sequence is a function that has as its domain the set of natural #'s

$$\begin{array}{cccccc} 1, & 2, & 3, & 4, & 5 & \ldots n \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 2, & 4, & 8, & 16, & 32 & \ldots 2^n \ldots \end{array}$$

what is the $n^{th}$ term of the sequence?

A finite sequence has as its domain $\{1, 2, \ldots, n\}$ for some finite n.

---

A series is the sum of the terms in a sequence.

$$a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{n} a_i$$

sigma notation

$\{2, 4, 8, 16, 32, \ldots\}$

$n^{th}$ term is $2^n$

the sum of the first 10 terms is

$$\sum_{i=1}^{10} 2^i = 2^1 + 2^2 + 2^3 + 2^4 + \ldots + 2^{10}$$

# Recursively-defined sequences/series

Fibonacci sequence:

1 2 3 4 5 6 7 8 9 10

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

$n^{th}$ term $a_n = a_{n-2} + a_{n-1}$

Recursively-defined sequence is dependent on previous terms

10.1

2. $a_n = (n-1)(n-2)(n-3)$

$a_1 = 0$

$a_2 = 0$

$a_3 = 0$

$a_4 = (4-1)(4-2)(4-3) = 3 \cdot 2 \cdot 1 = 6$

$a_5 = (5-1)(5-2)(5-3) = 4 \cdot 3 \cdot 2 = 24$

8. $a_n = (-1)^{n-1}(3n-5)$

$a_1 = (-1)^{1-1}(3\cdot 1-5) = -2$

$a_2 = (-1)^{2-1}(3\cdot 2-5) = -1$

$a_3 = (-1)^{3-1}(3\cdot 3-5) = 4$

$a_4 = (-1)^{4-1}(3\cdot 4-5) = -7$

$a_5 = (-1)^{5-1}(3\cdot 5-5) = 10$

$(-1)^n$

$\Rightarrow$ alternating sequence

$a_n = \left(1+\frac{1}{n}\right)^n \longrightarrow e$

find the $n^{th}$ term:

26. $-2, 3, 8, 13, 18, \ldots$

1 2 3 4 5

what function of n takes $\{1,2,3,\ldots,n\ldots\}$ to this sequence?

$a_n = a_{n-1} + 5$ ← recursive definition

$a_n = 5n - 7$

$5, 10, 15, 20, 25, 30, \ldots$

$a_n = 5n$

28. $\sqrt{2}, 2, \sqrt{6}, 2\sqrt{2}, \sqrt{10}, \ldots$

$\sqrt{2\cdot 1}$ $\sqrt{2\cdot 2}$ $\sqrt{2\cdot 3}$ $\sqrt{2\cdot 4}$ $\sqrt{2\cdot 5}$

$a_n = \sqrt{2n}$

32. $\ln e^2, \ln e^3, \ln e^4, \ln e^5, \ldots$

$a_n = \ln e^{n+1} = n+1$

10.1

# 7, 9, 25-31 odd

## Arithmetic Sequences/Series:

**Definition:** A sequence is arithmetic if there exists a number $d$, called the **common difference**, such that $\boldsymbol{a_{n+1} = a_n + d}$ for any integer $n \geq 1$.

The **nth term** of an arithmetic sequence is given by
$\boldsymbol{a_n = a_1 + (n-1)d}$, for any integer $n \geq 1$.

The **sum of the first n terms** of an arithmetic sequence is given by
$$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$$

## Geometric Sequences/Series:

**Definition:** A sequence is geometric is there is a number $r$, called the **common ratio**, such that $\boldsymbol{\frac{a_{n+1}}{a_n} = r}$ , or $a_{n+1} = a_n r$ , for any integer $n \geq 1$.

The **nth term of a geometric sequence** is given by
$\boldsymbol{a_n = a_1 r^{n-1}}$ , for any integer $n \geq 1$.

The **sum of the first n terms** of a geometric sequence is given by
$\boldsymbol{S_n = \frac{a_1(1-r^n)}{1-r}}$ , for any $r \neq 1$.

When $|r| < 1$, the **limit or sum of an infinite geometric series** is given by
$$\boldsymbol{S_\infty = \frac{a_1}{1-r}}$$