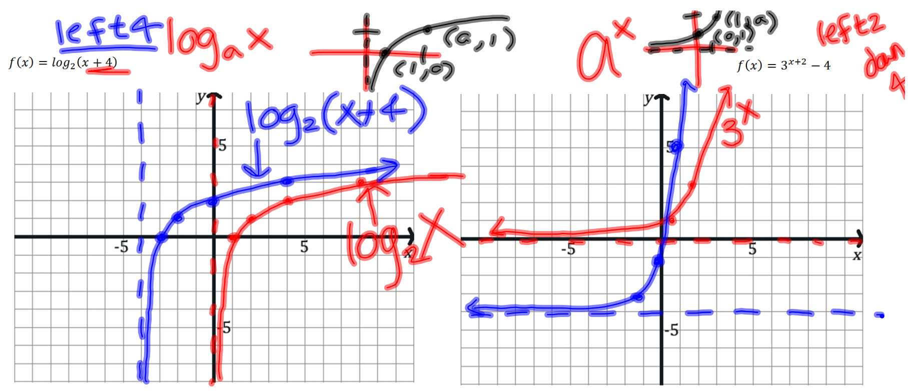
6. $\log_5 \frac{1}{125} = \boxed{-3}$ $\quad 5^x = \frac{1}{125} \quad \frac{1}{125} = \frac{1}{5^3} = 5^{-3}$

7. $\log_{27} 3 = \boxed{\frac{1}{3}}$ $\quad 27^x = 3 \quad \sqrt[3]{27} = 3 \quad 27^{1/3}$

8. $3^{2\log_3 2} = 3^{\log_3 2^2} = 3^{\log_3 4} = \boxed{4}$

9. $\ln \frac{e^2}{\sqrt[3]{e}} = \ln e^2 - \ln e^{1/3} = 2 - \frac{1}{3} = \frac{6}{3} - \frac{1}{3} = \boxed{\frac{5}{3}}$

10. $\log_4 8 = \frac{\log_2 8}{\log_2 4} = \boxed{\frac{3}{2}}$



14. Express as a single logarithm.

$4\ln x - 2[\ln 3y + 3\ln 2x]$

$P\log_a x = \log_a (x)^P$

$= 4\ln x - 2[\ln 3y + \ln(2x)^3]$

$= 4\ln x - 2\ln(3y \cdot 8x^3)$

$= \ln x^4 - \ln(24x^3y)^2$

$= \boxed{\ln \frac{x^4}{24^2x^6y^2}} = \boxed{\ln\left(\frac{1}{576x^2y^2}\right)}$

16. Solve for x.

$3^{5x+2} = \frac{1}{27}$

$3^{5x+2} = 3^{-3}$

$5x+2=-3$

$5x=-5$

$\boxed{x=-1}$

$\log_a b = c \Leftrightarrow a^c = b$

17. Solve for x.

$\log_3 x + \log_3(x-8) = 2$

$\log_3[x(x-8)] = 2$

$3^{\log_3[x(x-8)]} = 3^2$

$3^2 = x(x-8)$

$9 = x^2 - 8x$

$x^2-8x-9=0$

$(x-9)(x+1)=0$

$\boxed{x=9}$, $\cancel{x=-1}$

18. A certain element has a half-life of 30 years and we start with a 4000 gram sample of the element.

$P(30) = \frac{1}{2}P_0$ $\quad P(t)=P_0e^{kt}$

$\frac{1}{2}\cancel{P_0} = \cancel{P_0}e^{k\cdot 30}$ $\quad 2000=4000e^{kt}$

$\frac{1}{2}=e^{k\cdot 30}$

$\ln\frac{1}{2} = k\cdot 30$ $\quad k=\frac{\ln\frac{1}{2}}{30}$; $P(t)=4000e^{\frac{\ln\frac{1}{2}}{30}t}$

c. Determine the number of years t it will take for there to be only 100 grams of the element left.
in terms of the natural log.

$\ln\frac{1}{2} = \ln e^{k\cdot 30}$

$= \log_e e^{k\cdot 30} = k\cdot 30$

what t gives us $P(t)=100$

$\frac{100}{4000} = \frac{4000e^{kt}}{4000}$

$\frac{1}{40} = e^{kt}$

$\ln\frac{1}{40} = kt$

$\boxed{t = \frac{\ln\frac{1}{40}}{\left(\frac{\ln\frac{1}{2}}{30}\right)}}$

$\frac{\ln\frac{1}{40}}{\frac{1}{30}\ln\frac{1}{2}} = \frac{\ln(40)^{-1}}{\frac{1}{30}\cdot\ln 2^{-1}} = \frac{-\ln 40}{-\ln 2} \cdot 30 = \boxed{\frac{30\ln 40}{\ln 2}}$

10.1 Sequences and Series

An infinite sequence is a function that has as its domain the set of natural numbers.

$$\begin{array}{ccccccccccccc} 1 & , & 2 & , & 3 & , & 4 & , & 5 & , & \ldots & n & ,\ldots \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & & & \\ 2 & , & 4 & , & 8 & , & 16 & , & 32 & , & \ldots, & 2^n & ,\ldots \end{array}$$

What is the n^{th} term?

A finite sequence has as its domain $\{1, 2, 3, \ldots, n\}$ for some n

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A series is a sum of terms of a sequence

$$a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{n} a_i$$

"sigma notation"

HW

#7, 9, 23-31 odd

$2^1 + 2^2 + 2^3 + 2^4 + 2^5$

$2 + 4 + 8 + 16 + 32 = \sum_{i=1}^{5} 2^i$

n^{th} term $= 2^n$

$2+4+8+\ldots = \sum_{i=1}^{\infty} 2^i$

$\sum_{i=7}^{234} 2^i + \sum_{i=50,000}^{50,025} 2^i$

10.1

2. $a_n = (n-1)(n-2)(n-3)$

$a_1 = 0$

$a_2 = 0$

$a_3 = 0$

$a_4 = (4-1)(4-2)(4-3) = 3 \cdot 2 \cdot 1 = 6$

$a_5 = (5-1)(5-2)(5-3) = 4 \cdot 3 \cdot 2 = 24$

Fibonacci Sequence

$$\begin{array}{cccccccccc} 1, & 1, & 2, & 3, & 5, & 8, & 13, & 21, & 34, & \ldots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & ,\ldots \end{array}$$

Recursively-defined sequence defines an element of the sequence interms of other elements.

$$F_n = F_{n-1} + F_{n-2}$$

8. $a_n = (-1)^{n-1}(3n-5)$

alternating sequence

$(-1)^n$

$a_1 = (-1)^{1-1}(3 \cdot 1 - 5) = -2$

$a_2 = (-1)^{2-1}(3 \cdot 2 - 5) = -1$

$a_3 = (-1)^{3-1}(3 \cdot 3 - 5) = 4$

$a_4 = (-1)^{4-1}(3 \cdot 4 - 5) = -7$

$a_5 = (-1)^{5-1}(3 \cdot 5 - 5) = 10$

$$a_n = \left(1 + \frac{1}{n}\right)^n \rightarrow e$$

26. $-2, 3, 8, 13, 18, \ldots$

what is the n^{th} term?

$\cancel{a_n = n+5}$

$a_n = 5n - 7$

$5, 10, 15, 20, 25, \ldots$

$5\cdot1 \quad 5\cdot2 \quad 5\cdot3 \quad 5\cdot4 \quad 5\cdot5$

$5n$

32. $\ln e^2, \ln e^3, \ln e^4, \ln e^5, \ldots$

$a_n = n+1 = \ln e^{n+1}$

Arithmetic Sequences/Series:

Definition: A sequence is arithmetic if there exists a number d, called the **common difference**, such that $\boldsymbol{a_{n+1} = a_n + d}$ for any integer $n \geq 1$.

The **nth term** of an arithmetic sequence is given by
$\boldsymbol{a_n = a_1 + (n-1)d}$, for any integer $n \geq 1$.

The **sum of the first n terms** of an arithmetic sequence is given by
$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequences/Series:

Definition: A sequence is geometric is there is a number r, called the **common ratio**, such that $\frac{a_{n+1}}{a_n} = r$, or $a_{n+1} = a_n r$, for any integer $n \geq 1$.

The **nth term of a geometric sequence** is given by
$\boldsymbol{a_n = a_1 r^{n-1}}$, for any integer $n \geq 1$.

The **sum of the first n terms** of a geometric sequence is given by
$S_n = \frac{a_1(1-r^n)}{1-r}$, for any $r \neq 1$.

When $|r| < 1$, the **limit or sum of an infinite geometric series** is given by
$$S_\infty = \frac{a_1}{1-r}$$