$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 5 & 6 & 7 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + 2y + 3z = 4$$
$$5y + 6z = 7$$
$$5y = 7 - 6z$$
$$y = \frac{7}{5} - \frac{6}{5}z$$

$$\left(\quad , \frac{7}{5} - \frac{6}{5}z, \ z \right)$$

$$(x, f(x), g(x))$$
$$(f(y), y, g(y))$$
$$(f(z), g(z), z)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 5 & 6 & 8 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$0 = 1$$

10.1

write sigma notation. $\sum_{i=1}^{n} a_n$

56. $7+14+21+28+35+\ldots$

n^{th} term: $7n$

$$\sum_{i=1}^{\infty} 7i = \boxed{\sum_{n=1}^{\infty} 7n}$$

60. $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}$

$$\sum_{n=1}^{5} n^{-2} = \sum_{n=1}^{5} \frac{1}{n^2}$$

62. $9-16+25+\ldots+(-1)^{n+1}n^2$

$$\sum_{i=3}^{n}(-1)^{i+1}\cdot i^2$$

64.

$$\frac{1}{1\cdot 2^2}+\frac{1}{2\cdot 3^2}+\frac{1}{3\cdot 4^2}+\frac{1}{4\cdot 5^2}+\ldots$$

$$=\sum_{i=1}^{\infty}\frac{1}{i\cdot(i+1)^2}$$

Arithmetic Sequences/Series:

Definition: A sequence is arithmetic if there exists a number d, called the **common difference**, such that $\boldsymbol{a_{n+1} = a_n + d}$ for any integer $n \geq 1$.

The **nth term** of an arithmetic sequence is given by
$\boldsymbol{a_n = a_1 + (n-1)d}$, for any integer $n \geq 1$.

The **sum of the first n terms** of an arithmetic sequence is given by
$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequences/Series:

Definition: A sequence is geometric is there is a number r, called the **common ratio**, such that $\frac{a_{n+1}}{a_n} = r$, or $a_{n+1} = a_n r$, for any integer $n \geq 1$.

The **nth term of a geometric sequence** is given by
$\boldsymbol{a_n = a_1 r^{n-1}}$, for any integer $n \geq 1$.

The **sum of the first n terms** of a geometric sequence is given by
$S_n = \frac{a_1(1-r^n)}{1-r}$, for any $r \neq 1$.

When $|r| < 1$, the **limit or sum of an infinite geometric series** is given by
$$S_\infty = \frac{a_1}{1-r}$$

10.2 Arithmetic Sequences & Series

$-8, -5, -2, 1, 4, \ldots$

common difference : 3

n^{th} term: $a_n = a_1 + (n-1)d$

$= -8 + (n-1)\cdot 3$

$= -8 + 3n - 3 = \boxed{3n - 11}$

12^{th} term: $3(12) - 11 = 36 - 11 = \boxed{25}$

$7, 4, 1, \ldots$

find the 17^{th} term.

$d = -3$

$a_n = a_1 + (n-1)d$

$a_{17} = 7 + (17-1)(-3)$

$= \boxed{-41}$

The sum of an aritmetic sequence is called an arithmetic series.

The sum of the first n terms is

$S_n = \frac{n}{2}(a_1 + a_n)$

$11 + 7 + 3 + \ldots$

$a_{14} = -41$

$a_n = a_1 + (n-1)d$

$a_{14} = 11 + (14-1)(-4)$

$= -41$

$S_{14} = \frac{14}{2}(11 + (-41))$

$= 7(-30) = \boxed{-210}$

34. $\sum_{K=5}^{20} 8K = ? = \underbrace{8\cdot5}_{1^{st}\text{ term}} + 8\cdot6 + \ldots + \underbrace{8\cdot20}_{16^{th}\text{ term}}$

$a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{16} = \frac{16}{2}(8\cdot5 + 8\cdot20)$

$= 8(40 + 160) = 8(200) = \boxed{1600}$

10.1 # 59, 63, 67

10.2 # 9, 15, 19, 21, 25, 29