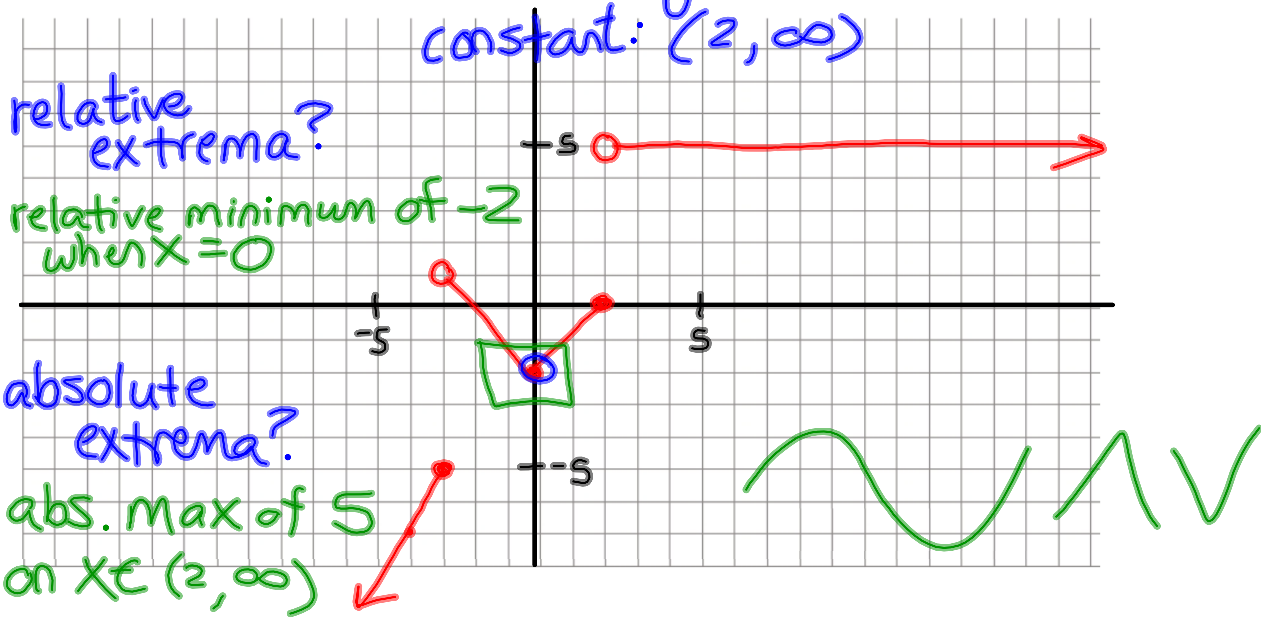
6. Graph the piecewise function and state the intervals over which the function is increasing, decreasing, or constant.

$$f(x) = \begin{cases} 2x+1, & x \le -3 \\ |x|-2, & -3 < x \le 2 \\ 5, & x > 2 \end{cases}$$

increasing: $(-\infty,-3) \cup (0,2)$

decreasing: $(-3,0)$

constant: $(2,\infty)$



relative extrema?

relative minimum of -2 when $x=0$

absolute extrema?

abs. max of 5 on $x \in (2,\infty)$

Homework questions?

10.2

29. Sum of multiples of 7 from 7 to 98, inclusive

$7 \cdot 1, 7 \cdot 2, 7 \cdot 3, \ldots, 7 \cdot 14$

$$\sum_{n=1}^{14} 7n$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{14} = \frac{14}{2}(7+98) = \boxed{735}$$

10.2

36. $\sum_{K=2}^{50}(2000-3k)$

$S_n=\frac{n}{2}(a_1+a_n) = \frac{49}{2}(1994+1850) = \boxed{94{,}178}$

$n=49$

$a_1=2000-3(2)=1994$

$a_{49}=2000-3(50)=1850$

Geometric Sequences/Series:

$\boxed{10.3}$

Definition: A sequence is geometric is there is a number r, called the **common ratio**, such that $\frac{a_{n+1}}{a_n}=r$, or $a_{n+1}=a_n r$, for any integer $n \geq 1$.

The **nth term of a geometric sequence** is given by
$a_n = a_1 r^{n-1}$, for any integer $n \geq 1$.

The **sum of the first n terms** of a geometric sequence is given by
$S_n = \frac{a_1(1-r^n)}{1-r}$, for any $r \neq 1$.

When $|r|<1$, the **limit or sum of an infinite geometric series** is given by
$$S_\infty = \frac{a_1}{1-r}$$

$18, -6, 2, \frac{-2}{3}$

common ratio: $-\frac{1}{3}$

$2, -10, +50, -250, \ldots$

find the 9th term:

$a_n = a_1 r^{n-1}$

$a_9 = 2 \cdot (-5)^{9-1} = \boxed{781{,}250}$

$16 - 8 + 4 - 2 + \ldots$

Find the sum of the first 10 terms.

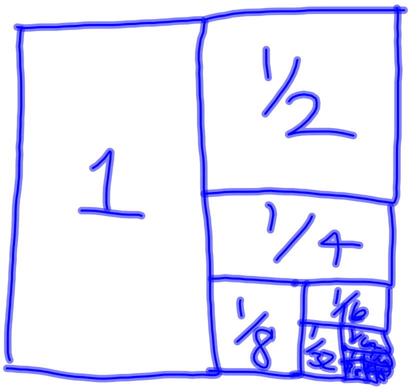
$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{16\left(1-\left(-\frac{1}{2}\right)^{10}\right)}{1-\left(-\frac{1}{2}\right)} = \frac{16\left(1-\frac{1}{2^{10}}\right)}{\frac{3}{2}}$$

$$= \frac{16\left(1-\frac{1}{1024}\right)}{\frac{3}{2}} = \boxed{\frac{341}{32} \approx 10.65625}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$$



$$= \boxed{2}$$

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \dots = 2$$

In general, when $|r| < 1$, the sum of an infinite geometric series is $S_\infty = \dfrac{a_1}{1-r}$

(or $S_\infty = \dfrac{a_0}{1-r}$ if series starts w/ a_0)

$$100 - 10 + 1 - \frac{1}{10} + \frac{1}{100} - \ldots$$

what is the sum of this infinite geometric series?

$$S_\infty = \frac{a_1}{1-r} = \frac{100}{1-\left(-\frac{1}{10}\right)}$$

$$= \frac{100}{\frac{10}{10}+\frac{1}{10}} = \frac{100}{\frac{11}{10}} = \boxed{\frac{1000}{11}}$$

50. $$\sum_{k=1}^{\infty} \frac{8}{3}\left(\frac{1}{2}\right)^{k-1}$$

$$S_\infty = \frac{a_1}{1-r}$$

$$r = \frac{1}{2}$$

$$a_1 = \frac{8}{3}\left(\frac{1}{2}\right)^{1-1} = \frac{8}{3}$$

$$a_2 = \frac{8}{3}\left(\frac{1}{2}\right)^{2-1} = \frac{4}{3}$$

$$a_3 = \frac{8}{3}\left(\frac{1}{2}\right)^{3-1} = \frac{2}{3}$$

$$S_\infty = \frac{8/3}{1-1/2} = \frac{8/3}{1/2} = \boxed{\frac{16}{3}}$$

Find Fraction Notation.

52. $0.222\ldots = 0.\overline{2}$

$0.2 + 0.02 + 0.002 + 0.0002 + \ldots$

$2 \cdot \frac{1}{10} + 2 \cdot \frac{1}{100} + 2 \cdot \frac{1}{1000} + 2 \cdot \frac{1}{10000} + \ldots$

$2 \cdot \frac{1}{10^1} + 2 \cdot \frac{1}{10^2} + 2 \cdot \frac{1}{10^3} + 2 \cdot \frac{1}{10^4} + \ldots$

$\sum_{n=1}^{\infty} \frac{2}{10^n}$ $\quad S_\infty = \frac{1/5}{1 - \frac{1}{10}} = \frac{\frac{1}{5}}{\frac{9}{10}} = \boxed{\frac{2}{9}}$

54. $6.161616\overline{16}$

$6.1 = 6.1 \times 10^0 = 6.1$

$+\ 0.061 = 6.1 \times 10^{-2} = 6.1 \cdot \frac{1}{100}$

$+\ 0.00061 = 6.1 \times 10^{-4} = 6.1 \cdot \frac{1}{100^2}$

$+\ 0.0000061 = 6.1 \times 10^{-6} = 6.1 \cdot \frac{1}{100^3}$

$$\sum_{n=0}^{\infty} \frac{6.1}{100^n} = \frac{6.1}{1 - \frac{1}{100}} = \frac{6.1}{\frac{99}{100}} = \boxed{\frac{610}{99}}$$

10.3

58. Someone offers you a job for the month of February (28 days). You will be paid \$0.01 the 1st day, \$0.02 the 2nd day, \$0.04 the 3rd day, doubling your previous day's salary each day.

$$S_{28} = \frac{a_1(1-r^n)}{1-r} = \frac{0.01(1-2^{28})}{1-2} = \$2,684,355$$

10.2

#35, 37

10.3

#15, 19, 21, 25, 35, 37, 43, 45, 49, 57