For the polynomial $p(x) = -3x^3(x-2)^2(x+3)(x-5)$

1. List the zeros and their corresponding multiplicities.

zeros: 0 2 -3 5

mult: 3 2 1 1

bounce

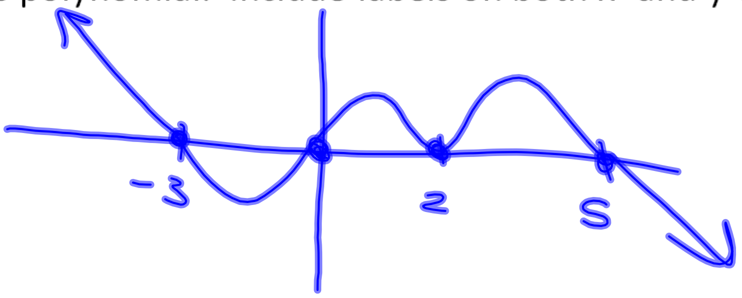
2. State the y-intercept as an ordered pair $(0, y)$.

(0, 0)

3. State the lead term and make a sketch to indicate what the Lead Term Test tells us about the end behavior of the graph.

$-3x^3 \cdot x^2 \cdot x \cdot x = -3x^7$

4. Graph the polynomial. Include labels on both x- and y-axes.




Homework questions?

10.3

25. $\frac{1}{18} - \frac{1}{6} + \frac{1}{2} - \dots$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_9 = \frac{\frac{1}{18}\left(1-(-3)^9\right)}{1-(-3)} = \frac{4921}{18}$$

10.2

35. $\sum_{k=0}^{19} \frac{k-3}{4}$

$S_n = \frac{n}{2}(a_1 + a_n)$

$n = 20$

$a_1 = \frac{0-3}{4} = \frac{-3}{4}$

$a_n = \frac{19-3}{4} = \frac{16}{4} = 4$

$= \frac{20}{2}\left(\frac{-3}{4} + 4\right)$

$= \boxed{\frac{65}{2}}$

10.3

If $|r|<1$, $S_\infty = \frac{a_1}{1-r}$

35. $25 + 20 + 16 + \ldots$

$r = \frac{20}{25} = \frac{16}{20} = \frac{4}{5}$

$S_\infty = \frac{25}{1-\frac{4}{5}}$

$= \frac{25}{1/5} = 25(5) = \boxed{125}$

37. $8 + 40 + 200 + \ldots$

$r = \frac{40}{8} = \frac{200}{40} = 5$

$= \infty$

does not exist

# 10.7 The Binomial Theorem

Expansion of $(a+b)^n$

$(a+b)^0 = 1$

$(a+b)^1 = a + b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

What patterns do you see?

exp. on a decreases from n to 0
b increases from 0 to n
n is coefficient of 2nd & 2nd to last terms
Sum of of all coefficients is $2^n$
sum of exponents in any term is n
pattern of coefficients follows Pascal's △

## Pascal's triangle

The first 21 rows

nth row of Pascal's △ corresponds to $(a+b)^{n-1}$

1 — $(a+b)^0$
1 1 — $(a+b)^1$
1 2 1 — $(a+b)^2$
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1 19 171 969 3876 11628 27132 50388 75582 92378 92378 75582 50388 27132 11628 3876 969 171 19 1
1 20 190 1140 4845 15504 38760 77520 125970 167960 184756 167960 125970 77520 38760 15504 4845 1140 190 20 1

Expand $(x-1)^4 = (x+(-1))^4$

$= 1 \cdot 1 \cdot x^4 + 4(-1)^1 x^3 + 6(-1)^2 x^2 + 4(-1)^3 x + 1(-1)^4 \cdot 1$

$= \boxed{x^4 - 4x^3 + 6x^2 - 4x + 1}$

8. $(2x-3y)^5$

$= 1(2x)^5 \cdot 1 + 5(2x)^4 \cdot (-3y) + 10(2x)^3 \cdot (-3y)^2 + 10(2x)^2(-3y)^3 + 5(2x)(-3y)^4 + 1 \cdot 1 (-3y)^5$

$= \boxed{32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5}$

## Binomial Coefficients

$\binom{n}{k}$ = "n choose k"

= the total # of combinations of n objects taken k at a time

"n factorial"

$n! = n(n-1)(n-2)\cdots 1$

$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

Note: $0! = 1$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Given a class of 22 students, how many groups of 7 can we make?

$$\binom{22}{7} = \frac{22!}{7!(22-7)!} = \frac{22!}{7!\,15!}$$

$$= \frac{22\cdot 21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16\cdot \cancel{15!}}{7!\cdot \cancel{15!}}$$

$$= \frac{\overset{11}{\cancel{22}}\cdot\cancel{21}\cdot\cancel{20}\cdot 19\cdot\overset{3}{\cancel{18}}\cdot 17\cdot 16}{\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 11\cdot 19\cdot 3\cdot 17\cdot 16 = \boxed{170{,}544}$$

# The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

In particular,

The $(k+1)^{st}$ term of $(a+b)^n$ is

$$\binom{n}{k} a^{n-k} b^k$$

Find the $5^{th}$ term of $(p-2q)^9$

$(k+1)^{st}$ term of $(a+b)^n$: $\binom{n}{k} a^{n-k} b^k$

$n = 9$

$k = 4$ ($5^{th}$ term is the $(4+1)^{st}$)

$a = p$

$b = -2q$

$5^{th}$ term:

$$\binom{9}{4}(p)^{9-4}(-2q)^4$$

$$= \frac{9!}{4!(9-4)!} \cdot p^5 \cdot 16q^4$$

$$= \frac{9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6}^{2} \cdot \cancel{5!}}{\cancel{4} \cdot \cancel{3} \cdot 2 \cdot \cancel{5!}} \cdot 16p^5q^4$$

$$= 16 \cdot 9 \cdot 7 \cdot 2\, p^5q^4 = \boxed{2016p^5q^4}$$

Given a set with n objects, the # of subsets containing k elements is $\binom{n}{k}$,
so the total # of subsets of any size is $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n}$

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$$(1+1)^n = \binom{n}{0}1^n\cdot 1^0+\binom{n}{1}1^{n-1}\cdot 1^1+\binom{n}{2}\cdot 1^{n-2}\cdot 1^2+\dots+\binom{n}{n}1^0\cdot 1^n$$
$$=\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n}$$

$\Rightarrow$ $\boxed{\text{The total \# of subsets of a set with n elements is } 2^n}$

How many "words" can we write with the English alphabet, which has 26 letters?

$$2^{26}=\boxed{67{,}108{,}864}$$

HW: 10.7 #1, 7, 21, 27, 31-39 odd