For the polynomial $p(x) = -3x^3(x-2)^2(x+3)(x-5)$

1. List the zeros and their corresponding multiplicities.

zeros: 0 2 -3 5

mult: 3 2 1 1

bounce

2. State the y-intercept as an ordered pair $(0, y)$.

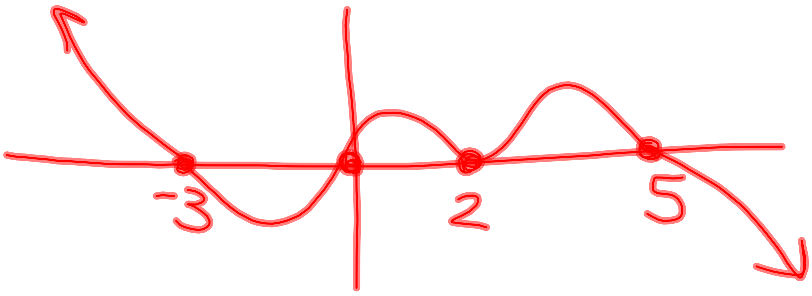
$(0,0)$

3. State the lead term and make a sketch to indicate what the Lead Term Test tells us about the end behavior of the graph.

$-3x^3 \cdot x^2 \cdot x \cdot x =$

$= -3x^7$

4. Graph the polynomial. Include labels on both x- and y-axes.



Homework questions?

10.2

35. $\sum_{k=0}^{19} \frac{k-3}{4}$

$n = 20$

$a_1 = \frac{0-3}{4} = \frac{-3}{4}$

$a_n = \frac{19-3}{4} = \frac{16}{4} = 4$

$S_n = \frac{n}{2}(a_1 + a_n)$

$= \frac{20}{2}\left(\frac{-3}{4} + 4\right)$

$= \boxed{\frac{65}{2}}$

10.3

If $|r|<1$, $S_\infty = \dfrac{a_1}{1-r}$

35. $25+20+16+\ldots$

$r = \dfrac{20}{25} = \dfrac{16}{20} = \dfrac{4}{5}$

$S_\infty = \dfrac{25}{1-\frac{4}{5}} = \dfrac{25}{1/5} = \boxed{125}$

37. $8+40+200+\ldots$

$r=5$ $S_\infty = \infty$ "does not exist"

15. $r=-25$

$a_1 = \dfrac{7}{625}$

$(-5)^2 \neq -5^2$

$a_{23} = a_1(r^{n-1})$

$= \dfrac{7}{625}(-25)^{23-1}$

$= \dfrac{7}{5^4}\cdot(5^2)^{22} = \dfrac{7}{5^4}\cdot 5^{44} = 7(5)^{40}$

10.7 The Binomial Theorem

Expansion of $(a+b)^n$

$(a+b)^0 = 1$

$(a+b)^1 = a + b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

What patterns do you see?

exponents on a decrease from n to 0
b increase from 0 to n
coefficients of 2nd & 2nd to last terms is n
sum of exponents of any term is n
$(a+b)^n$ has n+1 terms
Sum of all coefficients is 2^n
Coefficients follow Pascal's Triangle

Pascal's triangle

The first 21 rows

nth row of Pascal's △ corresponds to $(a+b)^{n-1}$

1 —— $(a+b)^0$
1 1 —— $(a+b)^1$
1 2 1 —— $(a+b)^2$
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1 19 171 969 3876 11628 27132 50388 75582 92378 92378 75582 50388 27132 11628 3876 969 171 19 1
1 20 190 1140 4845 15504 38760 77520 125970 167960 184756 167960 125970 77520 38760 15504 4845 1140 190 20 1

Expand $(x-1)^4 = (x+(-1))^4$

$a=x,\ b=-1,\ n=4$

$1x^4(1)+4x^3(-1)^1+6x^2(-1)^2+4x(-1)^3+1(1)(-1)^4$

$x^4-4x^3+6x^2-4x+1$

8. $(2x-3y)^5$

$a=2x$

$b=-3y$

$n=5$

$1(2x)^5(1)+5(2x)^4(-3y)+10(2x)^3(-3y)^2+10(2x)^2(-3y)^3+5(2x)(-3y)^4+1(1)(-3y)^5$

$\underbrace{32}_{2^5}x^5 \underbrace{-240}_{5\cdot 2^4(-3)}x^4y \underbrace{+720}_{10\cdot 2^3\cdot(-3)^2}x^3y^2 \underbrace{-1080}_{10\cdot 2^2\cdot(-3)^3}x^2y^3 \underbrace{+810}_{5\cdot 2\cdot(-3)^4}xy^4 \underbrace{-243}_{(-3)^5}y^5$

Binomial Coefficients

$\binom{n}{k}$ = "n choose k"

= the total # of combinations of n objects taken K at a time

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

"n factorial"

$n! = n(n-1)(n-2)\cdots 2\cdot 1$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

Given a class of 17 students, how many groups of 5 can we make?

$$\binom{17}{5} = \frac{17!}{5!(17-5)!} = \frac{17!}{5!\,12!}$$

$$= \frac{17\cdot \cancel{16}^{2}\cdot \cancel{15}\cdot 14\cdot 13\cdot \cancel{12!}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{12!}}$$

$$= 17\cdot 2\cdot 14\cdot 13 = \boxed{6188}$$

The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

In particular,
The $(k+1)^{st}$ term of $(a+b)^n$ is

$$\binom{n}{k} a^{n-k} b^k$$

Find the 5^{th} term of $(p-2q)^9$

$(k+1)^{st}$ term of $(a+b)^n$ is $\binom{n}{k} a^{n-k} b^k$

$a = p$

$b = -2q$

$n = 9$

$k = 4$

5th term is $(4+1)^{st}$ term

$$\binom{9}{4} p^{9-4} (-2q)^4$$

$$= \frac{9!}{4!(9-4)!} p^5 (16q^4)$$

$$= \frac{9 \cdot \cancel{8}^{2} \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{5!}} p^5 \cdot 16q^4$$

$$= 9 \cdot 2 \cdot 7 \cdot 16 p^5 q^4$$

$$= 2016 p^5 q^4$$

Given a set with n objects, the # of subsets containing k elements is $\binom{n}{k}$,
so the total # of subsets of any size is $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}$

$$(1+1)^n = \binom{n}{0}1^n\cdot 1^0+\binom{n}{1}1^{n-1}\cdot 1^1+\binom{n}{2}\cdot 1^{n-2}\cdot 1^2+\cdots+\binom{n}{n}1^0\cdot 1^n$$
$$=\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}$$

$\Rightarrow$ The total number of subsets of a set with n elements is 2^n.

How many "words" can we write with the English alphabet, which has 26 letters?

$$2^n = 2^{26} = \boxed{67{,}108{,}864}$$

HW: 10.7 # 1, 7, 21, 27, 31-39 odd