

$$y = \frac{1}{2}x - 3 \quad ; \quad (6, -1)$$

$m = -2$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -2(x - 6)$$

$$y + 1 = -2x + 12$$

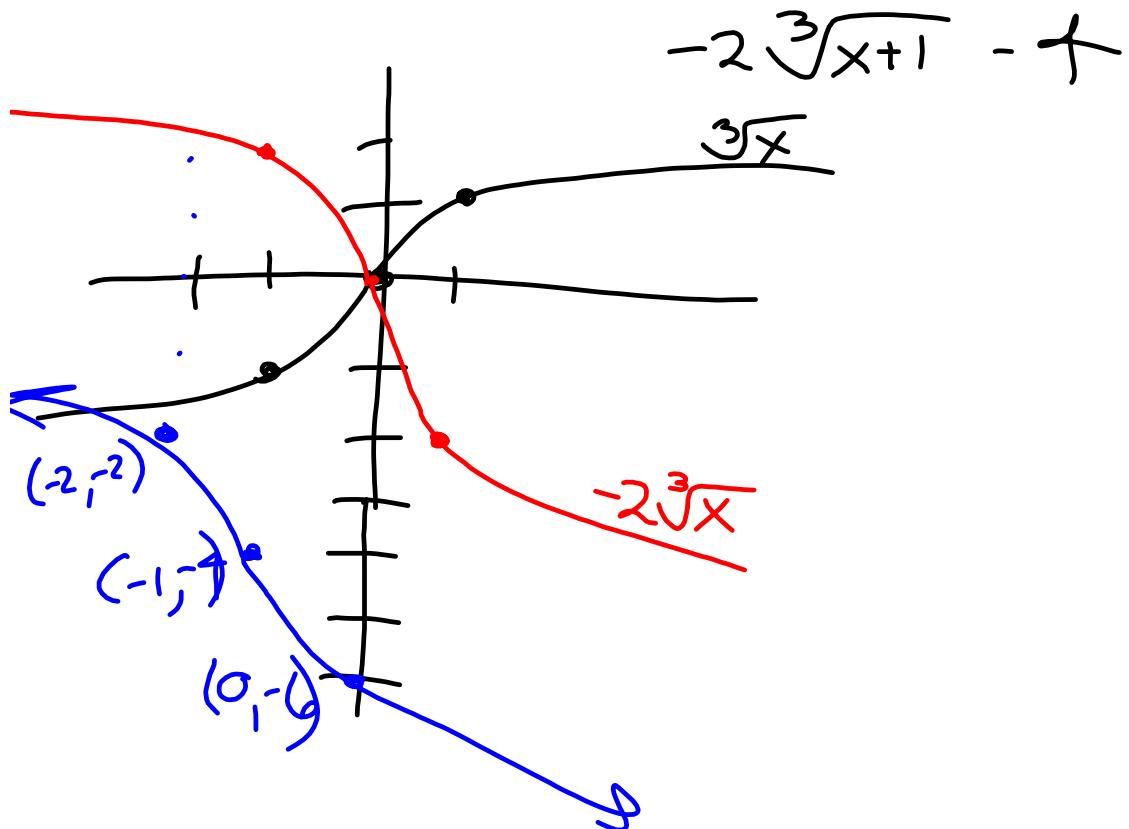
$$y = -2x + 11$$

$$\frac{f(x+h) - f(x)}{h}, \quad f(x) = x^2 - 2x - 4$$

$$\frac{(x+h)^2 - 2(x+h) - 4 - (x^2 - 2x - 4)}{h}$$

$$= \cancel{x^2 + 2xh + h^2} - \cancel{2x} - \cancel{2h} - \cancel{4} - \cancel{x^2} + \cancel{2x} + \cancel{4}$$

$$= \frac{h(2x + h - 2)}{h} = \boxed{2x + h - 2}$$



pt

3a.  $y = -(2x)^3 + 5$

b.  $y = \frac{3}{x+6} - 8 = 3 \cdot \frac{1}{x+6} - 8$

2.3 Quadratic Equations, Functions, and Models

A **quadratic equation** is an equation of the form:

$$ax^2 + bx + c = 0$$

A **quadratic function** is a function of the form:

$$f(x) = ax^2 + bx + c$$

$$y = ax^2 + bx + c$$

Equation-solving PrinciplesZero Product Property:

If  $MN=0$ , then  $M=0$  or  $N=0$

If  $(x-2)(x+3)=0$ , then  $x=2$  or  $x=-3$  are solutions.

$(x-2)(x+3)=5$  does NOT mean  $x-2=5$  or  $x+3=5$

Square Root Theorem:

If  $[f(x)]^2 = c$ , then  $f(x) = \pm\sqrt{c}$

If  $x^2 = 4$ , then  $x = \pm 2$

If  $(x-3)^2 = 16$ , then  $x-3 = \pm 4 \Rightarrow x = 7$  or  $x = -1$

The Quadratic Formula

If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$ax^2 + bx = -c$$

$$a(x^2 + \frac{bx}{a}) = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a}$$

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\frac{-c}{a} \cdot \frac{4a}{4a} = -\frac{4ac}{4a^2}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Discriminant

The discriminant is the  $b^2 - 4ac$  part of the quadratic formula.

If  $b^2 - 4ac > 0$ , the quadratic equation will have two distinct real roots (solutions).

If  $b^2 - 4ac = 0$ , the quadratic equation will have one real "double" root.

If  $b^2 - 4ac < 0$ , the quadratic equation will have two complex conjugate roots.

$\begin{cases} a+bi, \text{ where } a, b \in \mathbb{R}, i = \sqrt{-1} \\ a-bi \end{cases}$

$$f(x) = x^2$$

$$x^2 = 0$$

$$(x-0)(x-0) = 0$$

$$f(x) = 2x^2 - x + 3$$

$$a=2, b=-1, c=3$$

$$b^2 - 4ac = (-1)^2 - 1(2)(3)$$

$$= 1 - 24$$

$$= -23$$

$\Rightarrow f$  has 2 complex conjugate roots

$$f(x) = 2x^2 - x + 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{-23}}{2(2)} = \frac{1 \pm \sqrt{-23}}{4}$$

$$\frac{a}{b} \cdot \frac{c}{1}$$

$$= \frac{ac}{b \cdot 1}$$

$$= \frac{1 \pm i\sqrt{23}}{4}$$

$$= \boxed{\frac{1}{4} \pm \frac{\sqrt{23}}{4} i}$$

Equations Reducible to Quadratic

$$(a^n)^m = a^{nm}$$

94.  $y^6 - 26y^3 - 27 = 0$

Let  $u = y^3$

$$u^2 = (y^3)^2 = y^6$$

$$u^2 - 26u - 27 = 0$$

$$(u-27)(u+1) = 0$$

$$u = 27 \quad u = -1$$

$$y^3 = 27 \quad y^3 = -1$$

$$y = \sqrt[3]{27} \quad y = \sqrt[3]{-1}$$

$$\boxed{y = 3 \quad y = -1}$$

$$100. \quad x^{\frac{1}{2}} - 4x^{\frac{1}{4}} = -3$$

$$x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 3 = 0$$

$$(a^n)^m = a^{nm}$$

$$(a^{\frac{1}{n}})^2 = a^{\frac{2}{n}}$$

$$\text{Let } u = x^{\frac{1}{4}}$$

$$u^2 = (x^{\frac{1}{4}})^2 = x^{\frac{1}{2}}$$

~~$$(x^{\frac{1}{2}})^2 = x$$~~

$$(x^{\frac{1}{4}})^2 = x^{\frac{1}{2}}$$

$$u^2 - 4u + 3 = 0$$

$$(u-3)(u-1) = 0$$

$$u=3, u=1$$

$$x^{\frac{1}{4}} = 3, x^{\frac{1}{4}} = 1$$

$$(x^{\frac{1}{4}})^4 = 3^4, (x^{\frac{1}{4}})^4 = 1^4$$

$$x = 81, x = 1$$

**HW #3 (due Fri, 08/29)**

- 2.3: #1-19 odd      Solve quadratics by factoring and taking square roots
- #21-26 all      Determine zeros and x-intercepts given graph
- #35-53 odd      Solve using quadratic formula
- #55-60 all      Use discriminant to classify solutions
- #77-83 odd      Find zeros of a quadratic function
- #89, 93, 97, 99, 103      Solve equations reducible to a quadratic

**HW #4 (due Fri, 09/05)**

- 2.4: #3-13 odd      Find vertex, etc. and graph
- #29-37 odd      Find vertex, etc.
- #41-49 odd      Applications of quadratics
- 3.1
- 3.2