

2.4 - Analyzing Graphs of Quadratic Functions

Standard form: $f(x) = ax^2 + bx + c$

The graph of a quadratic function is a parabola.

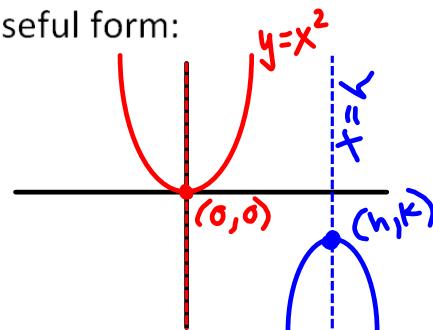
We can rewrite the standard form into a more useful form:

$$f(x) = a(x - h)^2 + k \quad , \text{ where}$$

Vertex: (h, k)

Axis of symmetry: $x = h$

(vertical line through the vertex)



If $a > 0$, parabola opens upward, vertex is a minimum

decreasing on $(-\infty, h)$; increasing on (h, ∞) ; range: $[k, \infty)$

If $a < 0$, parabola opens downward, vertex is a maximum

increasing on $(-\infty, h)$; decreasing on (h, ∞) ; range: $(-\infty, k]$

Domain: $(-\infty, \infty)$

12. $f(x) = 2x^2 - 10x + 14$ Completing
the Square

Step 1: factor x^2 -coefficient out of x^2 & x terms only

$$f(x) = 2(x^2 - 5x) + 14$$

Step 2: take $\frac{1}{2}$ of x -coefficient and square it

$$\frac{1}{2}(-5) = \frac{-5}{2}$$

$$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$

Step 3: add that number inside the parentheses, and subtract that number times any x^2 -coeff that was factored out previously outside the parentheses

$$f(x) = 2(x^2 - 5x + \left(\frac{-5}{2}\right)^2) + 14 - \left(\frac{25}{4}\right)(2)$$

Step 4: rewrite perfect square trinomial in parentheses as $(x-h)^2$ and combine constants outside the function

$$f(x) = 2\left(x - \frac{5}{2}\right)^2 + \frac{28}{2} - \frac{25}{2}$$

$$f(x) = 2\left(x - \frac{5}{2}\right)^2 + \frac{3}{2}$$

vertex: $\left(\frac{5}{2}, \frac{3}{2}\right)$

$$14. f(x) = -3x^2 - 3x + 1$$

$$f(x) = -3(x^2 + x) + 1$$

$$\frac{1}{2}(1) = \frac{1}{2}$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$f(x) = -3\left(x^2 + x + \left(\frac{1}{2}\right)^2\right) + 1 - \left(\frac{1}{4}\right)(-3)$$

$$f(x) = -3\left(x + \frac{1}{2}\right)^2 + \frac{7}{4}$$

$$\text{Vertex : } \left(-\frac{1}{2}, \frac{7}{4}\right)$$

$$f(x) = ax^2 + bx + c \quad \text{VERSUS} \quad f(x) = a(x - h)^2 + k$$

useful to determine :

- y-intercept
(0, c)

- x-intercept/zeros
(x, 0)

by factoring or
applying quadratic
formula

useful to determine :

- vertex (h, k)

- range $(-\infty, k] \text{ or } [k, \infty)$

-  & max/min

- axis of symmetry

zeros of a function v. x-intercepts of a function

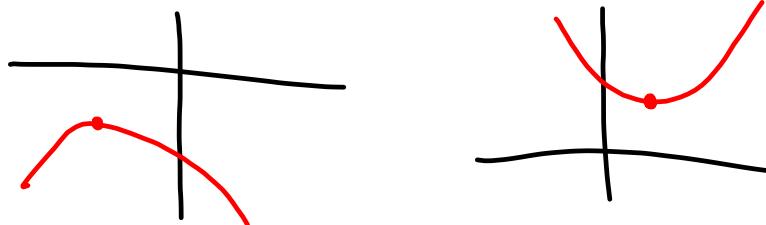
x is a zero of a function f if $f(x)=0$

That is, zeros of a function are all of the input values that have 0 as their output.

$(x,0)$ is an x-intercept of a function f if the graph of f intersects the x -axis at the point $(x,0)$.

Since the x -coordinates of the x -intercepts are exactly those input values that map to 0, we can get our x -intercepts from our list of zeros.

Note, however, that *only real zeros contribute to x-intercepts*.



$$f(x) = 3x^2 - x + 5$$

$$3x^2 - x + 5 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{1-60}}{6} = \frac{1 \pm \sqrt{-59}}{6}$$

$$f(x) = 3\left(x^2 - \frac{1}{3}x\right) + 5$$

$$= 3\left(x^2 - \frac{1}{3}x + \left(\frac{1}{6}\right)^2\right) + 5 - \left(\frac{1}{36}\right)(3)$$

$$f(x) = 3\left(x - \frac{1}{6}\right)^2 + \frac{59}{12}$$

vertex: $\left(\frac{1}{6}, \frac{59}{12}\right)$

axis of symmetry: $x = \frac{1}{6}$

y-intercept: $(0, 5)$

x-intercept(s): none

domain: $(-\infty, \infty)$

range: $\left[\frac{59}{12}, \infty\right)$

max/min? $\nearrow \frac{59}{12}$

increasing on:

$\left(\frac{1}{6}, \infty\right)$

decreasing on:
 $(-\infty, \frac{1}{6})$

$$f(x) = -2x^2 + x - 4$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-2)(-7)}}{2(-2)}$$

$$= \frac{-1 \pm \sqrt{1 - 32}}{-4} = \frac{-1 \pm \sqrt{-31}}{-4}$$

$$f(x) = -2(x^2 - \frac{1}{2}x) - 4$$

$$= -2(x^2 - \frac{1}{2}x + (\frac{1}{4})^2) - 4 - (\frac{1}{16})(-2)$$

$$= -2(x - \frac{1}{4})^2 - \frac{31}{8}$$

vertex: $(\frac{1}{4}, -\frac{31}{8})$

axis of symmetry:

$$x = \frac{1}{4}$$

y-intercept:

$$(0, -4)$$

x-intercept(s):

none

domain:

$$(-\infty, \infty)$$

range: $(-\infty, -\frac{31}{8}]$

max/min?

increasing on:

$$(-\infty, \frac{1}{4})$$

decreasing on:

$$(\frac{1}{4}, \infty)$$

42. Height of a Rocket

$$s(t) = -16t^2 + 150t + 40$$

determine time @ which rocket reaches max height & find that max height.

vertex: $(t, s(t))$

↑
max height

vertex: $(\frac{-b}{2a}, f(\frac{-b}{2a}))$

$$\frac{-b}{2a} = \frac{-150}{2(-16)} = \frac{150}{32} = \frac{75}{16}$$

time @
which it
reaches
max height

$$s\left(\frac{75}{16}\right) = -16\left(\frac{75}{16}\right)^2 + 150\left(\frac{75}{16}\right) + 40$$

= [] max height

HW #4 (due Fri, 08/28)

- 2.3: #1-19 odd
 - #21-26 all
 - #35-53 odd
 - #55-60 all
 - #77-83 odd
 - #89,93,97,99,103
- Solve quadratics by factoring and taking square roots
- Determine zeros and x-intercepts given graph
- Solve using quadratic formula
- Use discriminant to classify solutions
- Find zeros of a quadratic function
- Solve equations reducible to a quadratic

HW #5 (due Fri, 09/05)

- 2.4: #3-13 odd
 - Find vertex, etc.
 - #41-49 odd
- Find vertex, etc. #29-37 odd
- Applications of quadratics
- 3.1:#8-14 all
 - #23-31 all
- Describing simple characteristics of polynomials
- Determining zeros & multiplicities from factored polynomials
- 3.2: #16,17,21,22,24,25,27,28 Graph polynomials that are already factored