Turn in HW #4

2.3:	#1-19 odd	Solve quadratics by factoring and taking square roots
	#21-26 all	Determine zeros and x-intercepts given graph
	#35-53 odd	Solve using quadratic formula
	#55-60 all	Use discriminant to classify solutions
	#77-83 odd	Find zeros of a quadratic function
	#89,93,97,99,103	Solve equations reducible to a quadratic

97. $m^{2/3} - 2m^{1/3} - 8 = 0$

Let $u = m^{1/3}$

$u^2 = (m^{1/3})^2 = m^{2/3}$

$u^2 - 2u - 8 = 0$

$(u-4)(u+1) = 0$

$u = 4$, $u = -1$

$m^{1/3} = 4$, $m^{1/3} = -1$

$m = 4^3$, $m = -1$

$m = 64$

42. Height of a Rocket

vertex: $(t, s(t))$

max height

$s(t) = -16t^2 + 150t + 40$

determine time @ which rocket reaches max height & find that max height.

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\frac{-b}{2a} = \frac{-150}{2(-16)} = \frac{150}{32} = \frac{75}{16}$

time @ which it reaches max height

$s\left(\frac{75}{16}\right) = -16\left(\frac{75}{16}\right)^2 + 150\left(\frac{75}{16}\right) + 40$

$= \boxed{}$ max height

$$f(x) = ax^2 + bx + c \longrightarrow a(x-h)^2 + k$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}$$

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c \cdot \frac{4a}{4a} - \left(\frac{b^2}{4a^{\cancel{2}}}\right)\cancel{a}$$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

vertex: $\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$

$$= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

48. maximizing profit

profit = revenue − cost

$$P(x) = R(x) - C(x)$$

x = # of units sold

find max profit & # of units that must be sold to yield max profit.

$R(x) = 5x$; $C(x) = 0.001x^2 + 1.2x + 60$

$$P(x) = 5x - (0.001x^2 + 1.2x + 60)$$

$$P(x) = -0.001x^2 + 3.8x - 60$$

Vertex: $\left(\frac{-3.8}{2(-0.001)}, P\left(\frac{-3.8}{2(-0.001)}\right)\right)$

$\frac{3.800}{0.002} = \boxed{1900 \text{ units must be sold to yield max profit}}$

$$P(1900) = -0.001(1900)^2 + 3.8(1900) - 60$$

$= \boxed{\$3550 \text{ max profit}}$

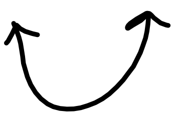
3.1/3.2 - Polynomial Functions

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

$a_n x^n$ is the lead term
a_n is the leading coefficient
n is the degree of the polynomial
a_0 is the constant term

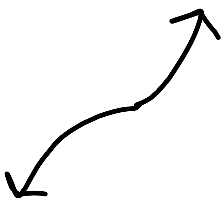
Is degree even or odd?

Every polynomial of even degree eventually behaves like $y = x^2$.



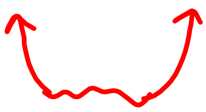
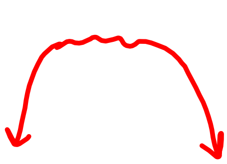
as $x \to \infty$, $x^2 \to \infty$
as $x \to -\infty$, $x^2 \to \infty$

Every polynomial of odd degree eventually behaves like $y = x^3$.



as $x \to \infty$, $x^3 \to \infty$
as $x \to -\infty$, $x^3 \to -\infty$

If leading coefficient is negative, vertical flip!

Degree:	Even	Odd
Leading Coeff: +		
-		

$$f(x) = 5x^4 - 3x^2 + 7$$

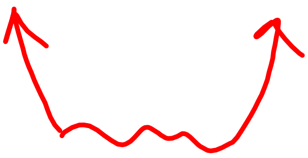
Lead term: $5x^4$

Leading coeff: 5

Degree: 4

Constant term: 7 $\Rightarrow$ y-int: (0,7)

end behavior:



The degree of a polynomial determines the number of zeros it has:

The Fundamental Theorem of Algebra

An n^{th} degree polynomial has n zeros (not necessarily unique), and can be written as the product of n linear factors.

$$f(x) = (x-b_1)(x-b_2)\cdots(x-b_n)$$

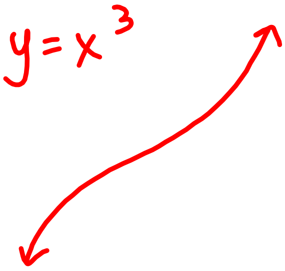
$$f(x) = x^5 = (x-0)(x-0)(x-0)(x-0)(x-0)$$

The graph of an n^{th} degree polynomial has at most n-1 turning points.

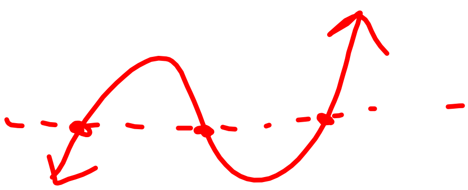
Examples:

cubic:

$y = x^3$



$y = ax^3 + bx^2 + cx + d$



6^{th} degree:

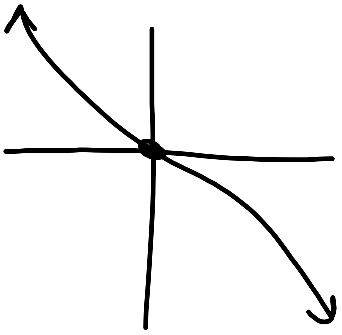
$f(x) = -2x^5 - x^3$

Lead term: $-2x^5$ $\Rightarrow$ End behavior:

constant term: 0

$\Rightarrow$ y-int: $(0,0)$

$-2x^5 - x^3 = -x^3(2x^2+1) = 0$



$-x^3 = 0$

$x^3 = 0$

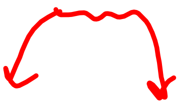
$x = 0$

$2x^2 + 1 = 0$

$2x^2 = -1$

$x^2 = -1/2 \Rightarrow x = \pm \frac{i}{\sqrt{2}}$

$y = -x^4 + 7x^3 - 6x^2$

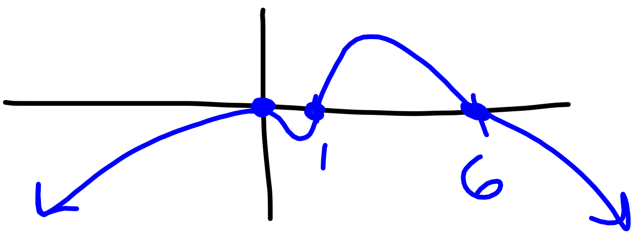
Lead term: $-x^4$ $\Rightarrow$ End behavior: 

Constant term: 0 $\Rightarrow$ y-int: $(0,0)$

$y = -x^2(x^2 - 7x + 6)$

$= -x^2(x-6)(x-1)$

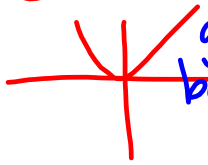
zeros: $0, 1, 6$



odd multiplicity:

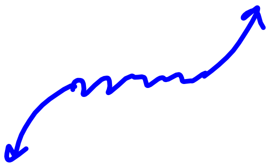
graph crosses through x-axis

even multiplicity:



graph bounces off x-axis

$y=(x-2)^4(x+3)^3$



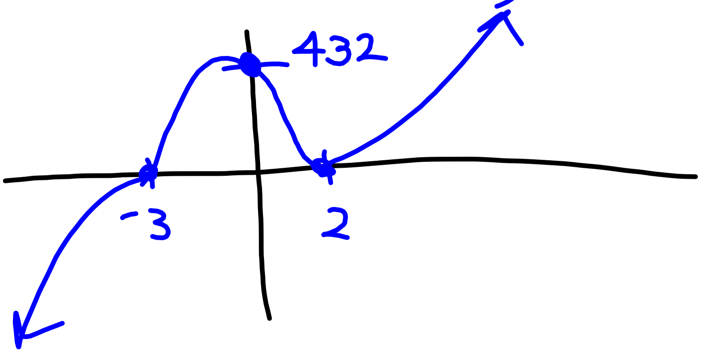
Lead term: $x^4 \cdot x^3 = x^7$

Constant term: $(-2)^4(3)^3 = 16 \cdot 27 = 432$

y-int: $(0, 432)$

zeros: 2 (even mult.)

-3 (odd mult.)



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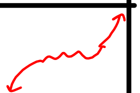
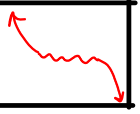
Lead Term Test

		Degree: Even	Odd
Leading Coefficient	+		
	-		

The degree of a polynomial determines the number of zeros it has:

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$$f(x) = (x-b_1)(x-b_2)\cdots(x-b_n)$$

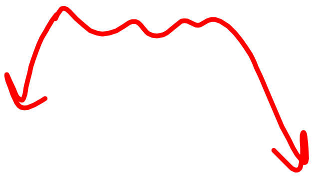
The graph of an n^{th} degree polynomial has at most n-1 turning points.

What is the end behavior of the polynomial?

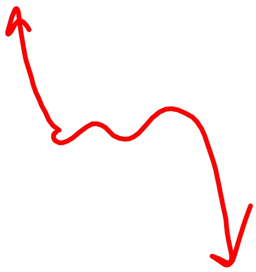
$y = -3x^4 + 2x^7 - 6$

$y = 15x^2 - 3x^8 + 4x^9 - 7$

$y = 7 - 2x^4 + 3x^3 - 15x$



$y = 22x + 3x^4 - x^5 + 1$



<u>HW #4</u> (submitted **Fri, 08/28**)

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<u>HW #5</u> (due **Fri, 09/05**)

- <u>2.4</u>: #3-13 odd — Find vertex, etc. and graph
  - #29-37 odd — Find vertex, etc.
  - #41-49 odd — Applications of quadratics
- <u>3.1</u>: #8-14 all — Describing simple characteristics of polynomials
  - #23-31 all — Determining zeros & multiplicities from factored polynomials
- <u>3.2</u>: #16,17,21,22,24,25,27,28 — Graph polynomials that are already factored