

Determine the lead term (end behavior), constant term (y-intercept), zeros (& their multiplicities), and graph the polynomial.

$$f(x) = -x^5 + 5x^4 - 6x^3 = -x^3(x^2 - 5x + 6)$$

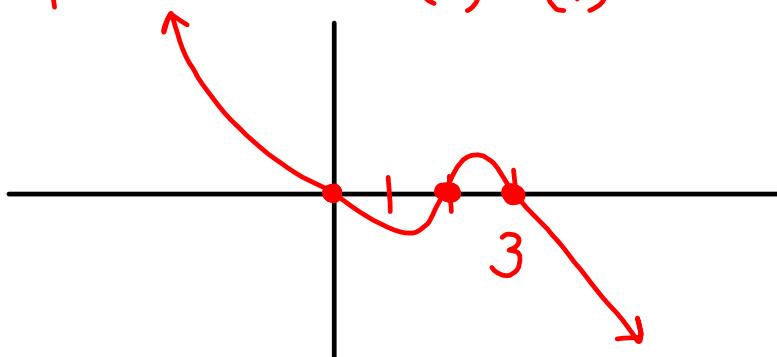
$$= -x^3(x-2)(x-3)$$

Lead term: $-x^5 \Rightarrow$ End behavior:

Constant term: 0 \Rightarrow y-intercept: (0,0)

zeros/
multiplicities:

(\circ)	$\frac{2}{(-)}$	$\frac{3}{(-)}$
-----------	-----------------	-----------------



Synthetic Division

12. $(x^3 - 7x^2 + 13x + 3) \div (x-2)$

$$\begin{array}{r} 2 | 1 & -7 & 13 & 3 \\ \downarrow & 2 & -10 & 6 \\ \hline 1 & -5 & 3 & 9 \end{array}$$

Quotient:

$$x^2 - 5x + 3$$

Remainder:

$$9$$

non-zero remainder
tells us that
 $x-2$ is not a factor
of $x^3 - 7x^2 + 13x + 3$

$$18. (x^7 - x^6 + x^5 - x^4 + 2) \div (x+1)$$

$$\begin{array}{r} -1 \\[-1ex] \boxed{-1} & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 2 \\[-1ex] \downarrow & & -1 & 2 & -3 & 4 & -4 & 4 & -4 \\[-1ex] \hline & 1 & -2 & 3 & -4 & 4 & -4 & 4 & \boxed{-2} \end{array}$$

Quotient:

$$x^6 - 2x^5 + 3x^4 - 4x^3 + 4x^2 - 4x + 4$$

Remainder:

$$-2$$

$$20. (x^5 + 32) \div (x+2)$$

$$\begin{array}{r} -2 \\[-1ex] \boxed{-2} & 1 & 0 & 0 & 0 & 0 & 32 \\[-1ex] \downarrow & & -2 & 4 & -8 & 16 & -32 \\[-1ex] \hline & 1 & -2 & 4 & -8 & 16 & \boxed{0} \end{array}$$

remainder: 0

$$\text{quotient: } x^4 - 2x^3 + 4x^2 - 8x + 16$$

$$x^5 + 32 = (x+2)(x^4 - 2x^3 + 4x^2 - 8x + 16)$$

$$32. f(x) = 3x^3 + 11x^2 - 2x + 8$$

are the #'s -4 & 2 zeros of the function?

$$\begin{array}{r} \underline{-4} \\ \begin{array}{rrrr} 3 & 11 & -2 & 8 \\ -12 & 4 & -8 \\ \hline 3 & -1 & 2 & \boxed{0} \end{array} \end{array} \quad \text{yes}$$

$$\begin{array}{r} \underline{2} \\ \begin{array}{rrrr} 3 & 11 & -2 & 8 \\ 6 & 34 & 64 \\ \hline 3 & 17 & 32 & \boxed{72} \end{array} \end{array} \quad \text{no}$$

$$f(x) = (x+4)(3x^2 - x + 2)$$

$$\begin{aligned} \text{zeros: } x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(2)}}{2(3)} \\ &= \frac{1 \pm \sqrt{-23}}{6} = \frac{1}{6} \pm \frac{\sqrt{23}}{6}i \end{aligned}$$

$$f(x) = (x - (-4))(x - (\frac{1}{6} + \frac{\sqrt{23}}{6}i))(x - (\frac{1}{6} - \frac{\sqrt{23}}{6}i))$$

$$46. f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$$

Factor the polynomial and solve $f(x) = 0$.

$$\begin{array}{r} \underline{1} \\ \begin{array}{rrrrrr} 1 & -4 & -7 & 34 & -24 \\ 1 & -3 & -10 & 24 \\ \hline 1 & -3 & -10 & 24 & \boxed{0} \end{array} \end{array}$$

$$f(x) = (x-1)(x^3 - 3x^2 - 10x + 24)$$

$$\begin{array}{r} \underline{2} \\ \begin{array}{rrrr} 1 & -3 & -10 & 24 \\ 2 & -2 & -24 \\ \hline 1 & -1 & -12 & \boxed{0} \end{array} \end{array}$$

$$f(x) = (x-1)(x-2)(x^2 - x - 12)$$

$$f(x) = (x-1)(x-2)(x-4)(x+3)$$

Zeros/Solutions to $f(x) = 0$:

$$\boxed{1, 2, 4, -3}$$

3.3

38. Determine if i or $-i$ are zeros of the polynomial $f(x) = x^3 + 2x^2 + x + 2$

$$\begin{array}{r} i \\ \downarrow \\ 1 \quad 2 \quad 1 \quad 2 \\ \hline 1 \quad 2+i \quad 2i \quad | 0 \end{array}$$

$\frac{2i+i^2}{2i^2} + (2i-1)$

If $a+bi$ is zero, so is $a-bi$

$\Rightarrow f(x)$ has both i & $-i$ as zeros

$\Rightarrow f(x)$ has $(x+i)(x-i) = x^2 - i^2 = x^2 + 1$ as a factor

$$\begin{array}{r} x+2 \\ x^2+1 \quad \overline{x^3+2x^2+x+2} \\ - (x^3+x) \\ \hline -2x^2+2 \\ - (2x^2+2) \\ \hline 0 \end{array}$$

$$f(x) = (x-i)(x+i)(x+2)$$

If the discriminant $b^2 - 4ac < 0$,

we know that the quadratic function has two complex conjugate zeros.

This means that if i is a zero, then so is $-i$, and in general,

if $a+bi$ is a zero, then so is $a-bi$.

If $f(x)$ has $a+bi$ and $a-bi$ as its only zeros, then

$$f(x) = [x - (a+bi)][x - (a-bi)]$$

also for $a+\sqrt{b}$ & $a-\sqrt{b}$

$a + \sqrt{b}$ and $a - \sqrt{b}$ also come in conjugate pairs.

$$-\sqrt{3} \quad 2+i$$

If 2, $\sqrt{3}$, and $2 - i$ are zeros of the polynomial $p(x)$, then

$$\begin{aligned} p(x) &= (x-2)(x-\sqrt{3})(x-(-\sqrt{3}))(x-(2-i))(x-(2+i)) \\ &= (x-2)(x-\sqrt{3})(x+\sqrt{3})(x-2+i)(x-2-i) \end{aligned}$$

Rational Zeros Theorem

Given a polynomial $p(x) = a_n x^n + \dots + a_1 x + a_0$,

The only possible rational zeros are of the form

$$\pm \frac{\text{factors of } a_0}{\text{factors of } a_n}$$

$$p(x) = 4x^3 - 7x^2 + 21x - 5$$

$\pm 1, \pm 2, \pm 4$ $\pm 1, \pm 5$

Possible rational zeros:

$$\frac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 4} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}$$

3.4

74. Find all the zeros of the polynomial $f(x) = 2x^3 + 3x^2 + 2x + 3$

possible rational zeros: $\frac{\pm 1, 3}{\pm 1, 2}$
 $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}$

$$\begin{array}{r} \frac{-1}{2} \\ \hline 2 & 3 & 2 & 3 \\ & -1 & -1 & -\frac{1}{2} \\ \hline 2 & 2 & 1 & \frac{5}{2} \end{array} \quad \ddots$$

$$\begin{array}{r} \frac{-3}{2} \\ \hline 2 & 3 & 2 & 3 \\ & -3 & 0 & -3 \\ \hline 2 & 0 & 2 & 0 \end{array}$$

$$f(x) = (x + \frac{3}{2})(2x^2 + 2)$$

$$2x^2 + 2 = 0$$

$$2x^2 = -2$$

$$x^2 = -1$$

$$x = \pm i$$

$$f(x) = 2(x + \frac{3}{2})(x - i)(x + i)$$

zeros: $-\frac{3}{2}, i, -i$

HW #5 (due Fri, 09/05)

- 2.4: #3-13 odd Find vertex, etc. and graph
#29-37 odd Find vertex, etc.
#41-49 odd Applications of quadratics
- 3.1: #8-14 all Describing simple characteristics of polynomials
#23-31 all Determining zeros & multiplicities from factored polynomials
- 3.2: #16,17,21,22,24,25,27,28 Graph polynomials that are already factored
- 3.1: #32-38 all Finding zeros & multiplicities of expanded polynomials
- 3.2: #18-20 all, 23, 26, 29-32 all Graphing expanded polynomials
- 3.3: #7, 9, 13, 19, 21, 23, 35 Long & synthetic polynomial division

HW #6 (due Fri, 09/12)

- 3.4: #7-16 all Given the zeros of a polynomial, find the polynomial
#25-32 all; 43-47 odd Given some zeros of a polynomial, find the other zeros
#51-54 all List all possible rational zeros
#55-69 odd Find all the zeros and write $f(x)$ in factored form
#79, 89, 93 Descartes' rule of signs
#95-98 all Graph the polynomial
- 3.5