3.4

Graph the polynomial.

96. $f(x) = 3x^3 - 4x^2 - 5x + 2$

$-1 \mid 3 \quad -4 \quad -5 \quad 2$

$\quad\quad\quad -3 \quad 7 \quad -2$

$\quad 3 \quad -7 \quad 2 \quad 0$

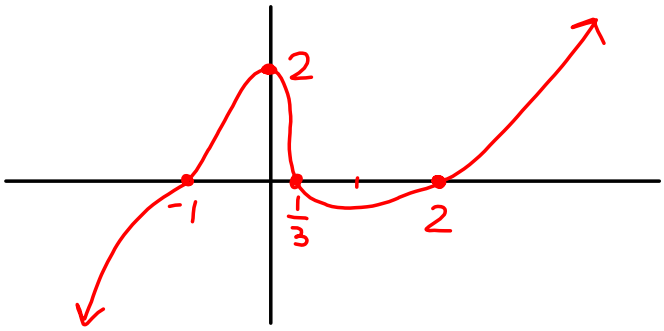
lead term: $3x^3$

y-int: $(0,2)$

$f(x) = (x+1)(3x^2 - 7x + 2)$

$= (x+1)(3x-1)(x-2)$

zeros: $-1, \frac{1}{3}, 2$ (all mult. 1)




3.4

Graph the polynomial.

98. $f(x) = 3x^4 - 37x^2 + 9$

lead term: $3x^4$

y-int: $(0,9)$

Let $u = x^2$

$u^2 = x^4$

$= 3u^2 - 37u + 9$

$u = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(3)(9)}}{2(3)}$

$42$

$37$

$37$

$259$

$1110$

$1369$

$-108$

$1261$

$x^2 = u = \frac{37 \pm \sqrt{1261}}{6}$

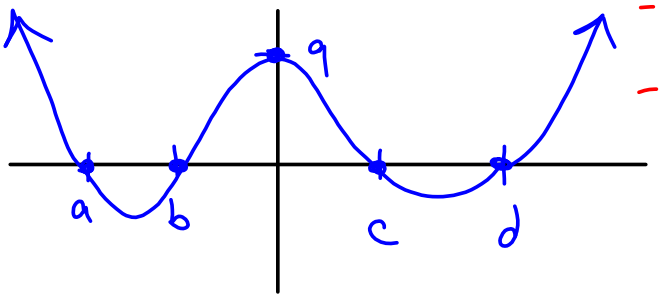
$x = \pm\sqrt{\frac{37 \pm \sqrt{1261}}{6}}$

zeros: $\sqrt{\frac{37+\sqrt{1261}}{6}} = d$

(all mult. 1) $\sqrt{\frac{37-\sqrt{1261}}{6}} = c$

$-\sqrt{\frac{37-\sqrt{1261}}{6}} = b$

$-\sqrt{\frac{37+\sqrt{1261}}{6}} = a$



**3.5 – Rational Functions** – Fractions made of polynomials!

$$f(x) = \frac{p(x)}{q(x)}$$

**$y$-intercept**: $(0, f(0))$
plug 0 in for $x$, unless $f(0)$ is undefined, in which case there is no $y$-intercept

**$x$-intercept(s)/zeros**: solutions to $p(x) = 0$
an entire fraction $= 0$ when the numerator $= 0$

When the denominator is zero $(q(x) = 0)$, $f(x)$ is undefined.

If a particular $x$-value makes both the numerator and denominator equal to 0, that factor will cancel and we will have a **hole** in the graph for that $x$-value.

If an $x$-value makes only the denominator equal to 0, the graph will have a **vertical asymptote** at that $x$-value.

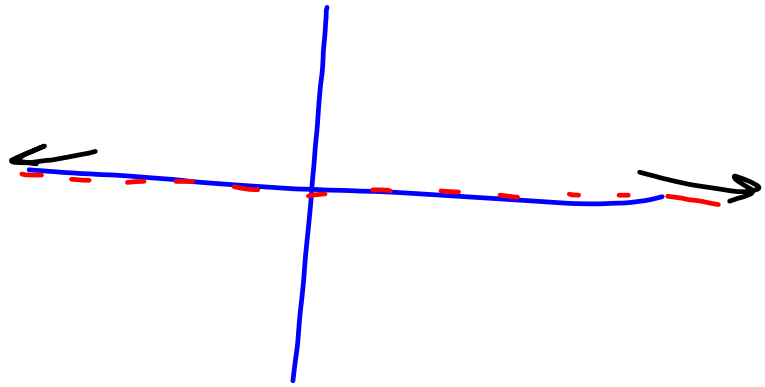
**End behavior** (what happens to $f(x)$ as $x \to \pm\infty$)

Look at the ratio of lead terms.

If the denominator has a larger degree than the numerator, then the graph will have a horizontal asymptote at $y = 0$.

$(f(x) \to 0$ as $x \to \pm\infty)$

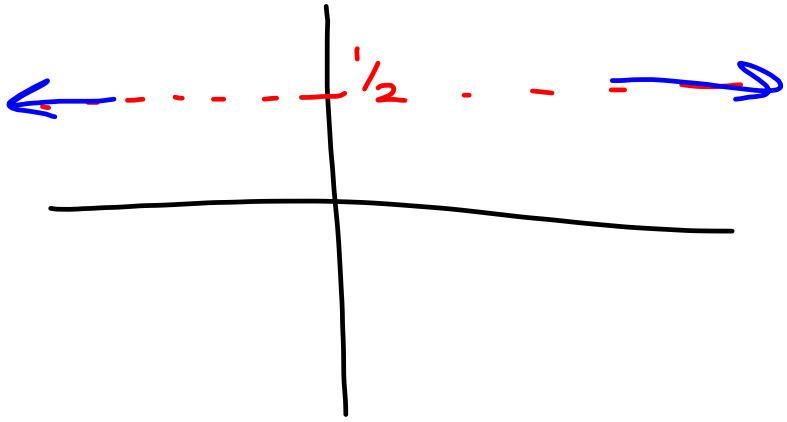
Example: $f(x) = \frac{-3x^5-2x^2+3}{5x^9+2x}$ $\approx \frac{-3x^5}{5x^9} = \frac{-3}{5x^4} \to 0$



End behavior, cont.

If the numerator and denominator have the same degree, then the graph will have a horizontal asymptote at $y = (the\ ratio\ of\ leading\ coefficients)$

Example: $f(x) = \frac{5x^2-4}{10x^2+3x} \approx \frac{5x^2}{10x^2} = \frac{1}{2}$




End behavior, cont.

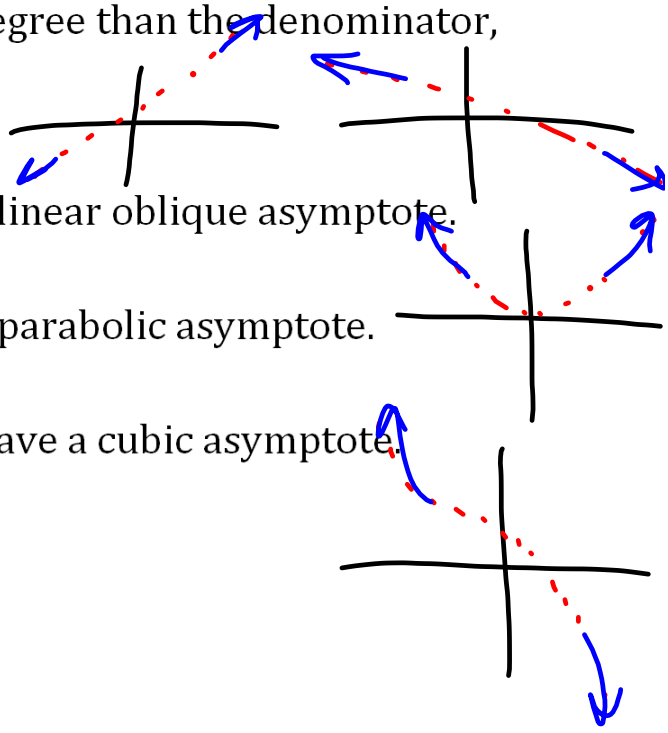
If the numerator has a larger degree than the denominator, perform long division.

If the degree is 1 higher,
the graph has a linear oblique asymptote.

If the degree is 2 higher,
the graph has a parabolic asymptote.

If the degree is 3 higher,
the graph will have a cubic asymptote.

Etc.



$$f(x) = \frac{x^2 - 7x + 6}{x + 5} = \frac{(x-6)(x-1)}{x+5}$$

$$x+5 \overline{)\, x^2 - 7x + 6}$$

$$-(x^2 + 5x)$$

$$-12x + 6$$

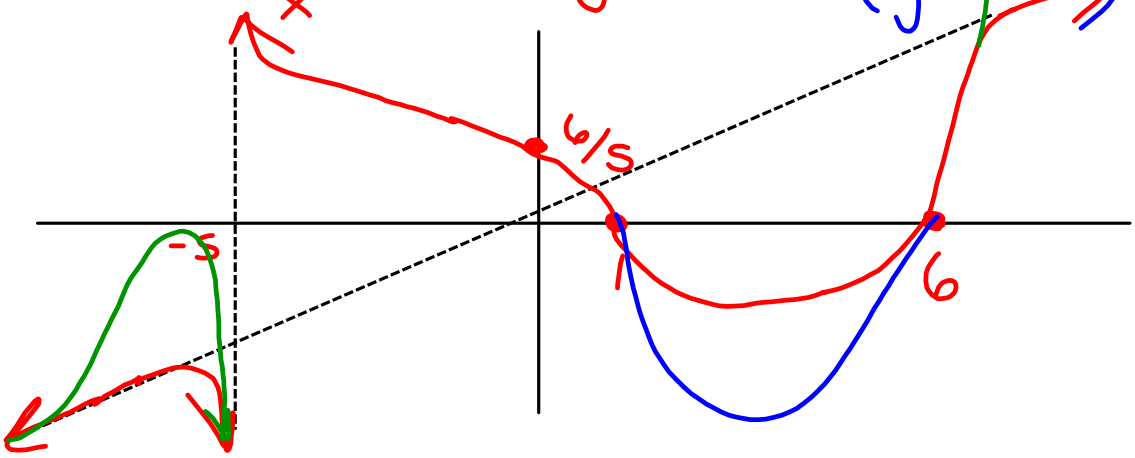
$$-(-12x - 60)$$

zero(s): 1 ; 6

y-intercept: $(0, \frac{6}{5})$

vertical asymptote(s): $x = -5$

end behavior: $\frac{x^2}{x} = x$ $y = x$ $(y = x - 12)$



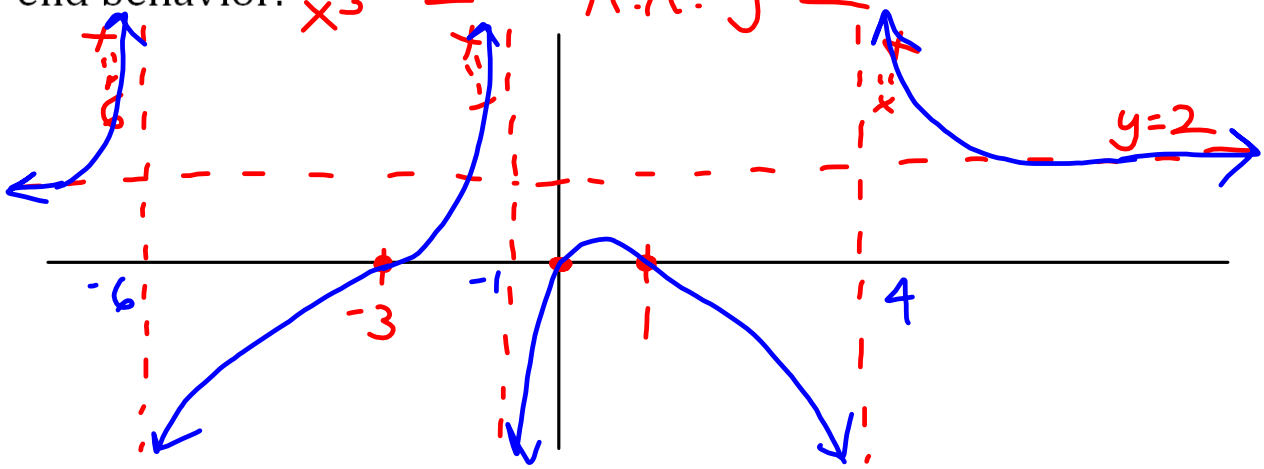

$$f(x) = \frac{2x(x+3)(x-1)}{(x-4)(x+1)(x+6)}$$

zero(s): 0, -3, 1

y-intercept: (0,0)

vertical asymptote(s): $x = 4, x = -1, x = -6$

end behavior: $\frac{2x^3}{x^3} = 2$ H.A. $y = 2$



**HW #6 (due Fri, 09/12)**

- 3.4: #7-16all Given the zeros of a polynomial, find the polynomial
  #25-32all; 43-47odd Given some zeros of a polynomial, find the other zeros
  #51-54all List all possible rational zeros
  #55-69odd Find all the zeros and write f(x) in factored form
  #79,89,93 Descartes' rule of signs
  #95-98all Graph the polynomial
- 3.5: #7-25odd Determining asymptotes of rational functions
  #27-67odd Graphing rational functions
- 3.6: #15-39odd Solving polynomial inequalities
  #47, 53-61odd Solving rational inequalities

**Quiz #3 - Wed, 09/10**
**Test #2 - Mon, 09/15**