

Turn in HW #6

- 3.4: #7-16all Given the zeros of a polynomial, find the polynomial
#25-32all; 43-47odd Given some zeros of a polynomial, find the other zeros
#51-54all List all possible rational zeros
#55-69odd Find all the zeros and write $f(x)$ in factored form
#79, 89, 93 Descartes' rule of signs
#95-98all Graph the polynomial
- 3.5: #7-25odd Determining asymptotes of rational functions
#27-67odd Graphing rational functions

Due Monday (if not submitted today):

- 3.6: #15-39odd Solving polynomial inequalities
#47, 53-61odd Solving rational inequalities

Test #2 - Mon, 09/15

1. Which of the following are not possible rational zeros of the polynomial $f(x) = 3x^5 - 4x^4 + 2x^3 - x - 4$?

(circle your answer(s)) $\frac{\text{factors of } 7}{\text{factors of } 3} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3}$

a. $\frac{2}{3}$

b. -1

c. 4

d. 3

e. -2

f. $-\frac{3}{2}$

2. Which of the following does Descartes' Rule of Signs tell us about the zeros of the polynomial

$f(x) = 3x^5 - 4x^4 + 2x^3 - x - 4$? (circle your answer(s))

3 sign changes \Rightarrow 3 or 1 positive real zeros

$$\begin{aligned} f(-x) &= 3(-x)^5 - 4(-x)^4 + 2(-x)^3 - (-x) - 4 \\ &= -3x^5 - 4x^4 - 2x^3 + x - 4 \end{aligned}$$

2 sign changes

a. it has 4 or 2 or 0 positive real zeros

d. it has 1 negative real zero

b. it has 3 or 1 positive real zeros

e. it has 2 or 0 negative real zeros

c. it has no negative real zeros

f. it has 3 or 1 negative real zero

3. Construct the polynomial of *least* degree that has the following as some of its zeros: 2 ; -5 ; $\sqrt{3}$; $7i$
 (Please leave your polynomial in factored form!)

$$f(x) = (x-2)(x+5)(x-\sqrt{3})(x+\sqrt{3})(x-7i)(x+7i)$$

4. Given the factored rational function $f(x) = \frac{x(x-4)(x+2)}{x-3}$

a. Determine its zero(s) if any:

$$0, 4, -2$$

b. Determine its vertical asymptote(s) if any:

$$x=3$$

5. Divide the polynomials using long division. State the quotient and the remainder. What is implied about the polynomial $x^4 - 2x^2 + 3$ and the number -2 by the remainder?

$$\begin{array}{r} x^4 - 2x^2 + 3 \\ \underline{- (x^4 + 2x^3)} \\ -2x^3 - 2x^2 + 3 \\ \underline{- (-2x^3 - 4x^2)} \\ 2x^2 + 3 \\ \underline{- (-4x - 8)} \\ 11 \end{array} \quad Q: x^3 - 2x^2 + 2x - 4 \quad R: 11$$

$\Rightarrow x+2$ is not a factor of $x^4 - 2x^2 + 3$
 -2 is not a zero of $x^4 - 2x^2 + 3$
 $f(-2) = 11 \leftarrow$ Remainder Theorem

6. For the polynomial function given below in factored form, determine

$$f(x) = -2\left(x - \frac{1}{3}\right)^2 (x-3)^3 (x+2)^2 \left(x + \frac{1}{2}\right)^3$$

A. The zeros of the function and their multiplicities:

Zero:	$\frac{1}{3}$	3	-2	$-\frac{1}{2}$			
Multiplicity:	2	3	2	3			

- B. The lead term of the polynomial and a sketch of what that implies for the end behavior of the graph:

$$-2 \cdot x^2 \cdot x^3 \cdot x^2 \cdot x^3 = -2x^{10}$$

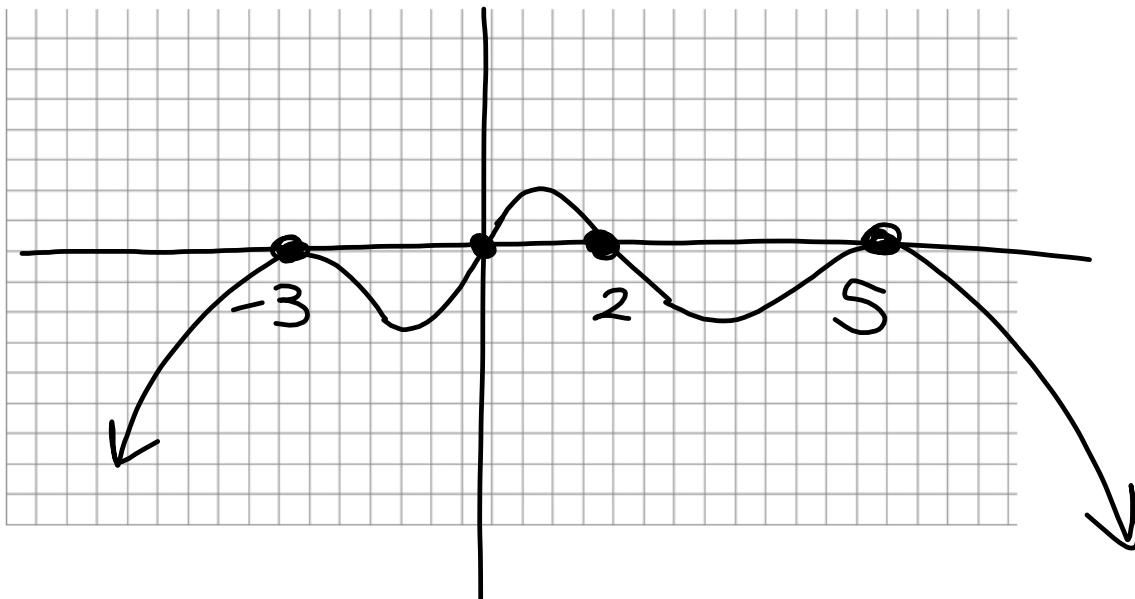
- C. The y-intercept of the function

$$-2\left(\frac{1}{3}\right)^2 (-3)^3 (2)^2 \left(\frac{1}{2}\right)^3$$

$$\frac{12}{1} \cdot \frac{1}{9} \cdot \frac{127}{1} \cdot \frac{4}{1} \cdot \frac{1}{8} = 3$$

7. Given the information about the polynomial function, draw its graph.
 Lead term: $-3x^8$

Zeros:	0	2	5	-3
Multiplicity:	3	1	2	2



8. Given that -2 is a zero of the polynomial $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$, find all other zeros and write the polynomial as a product of linear factors.

$$\begin{array}{r} \boxed{-2} \quad 1 \quad 5 \quad 5 \quad -5 \quad -6 \\ \quad \quad -2 \quad -6 \quad 2 \quad \quad 6 \\ \hline \quad 1 \quad 3 \quad -1 \quad -3 \quad \boxed{0} \end{array} \qquad \begin{array}{r} \boxed{-1} \quad 1 \quad 3 \quad -1 \quad -3 \\ \quad \quad -1 \quad -2 \quad \quad 3 \\ \hline \quad 1 \quad 2 \quad -3 \quad \boxed{0} \end{array}$$

$$\begin{aligned}
 f(x) &= (x+2)(x^3 + 3x^2 - x - 3) = (x+2)(x+1)(x^2 - 1) \\
 &= (x+2) \left[x^2(x+3) - 1(x+3) \right] = (x+2)(x+3)(x^2 - 1) \\
 &= (x+2)(x+3)(x-1)(x+1)
 \end{aligned}$$

zeros: $-2, -3, 1, -1$

9. Describe how the following transformations affect the graph of the original untransformed function (e.g. "shift up 2" or "stretch vertically by a factor of 3 and flip upside down" etc.):

A. The +3 in $f(x) = |x + 3|$

shift left 3

B. The 5 in $f(x) = 5\sqrt[3]{x}$

vertical stretch by 5

C. The - in $f(x) = -x^2$

flip upside down (vertically)

D. The -4 in $f(x) = \frac{1}{x} - 4$

shift down 4

3.6

64. Solve the inequality.

$$\frac{2x}{x^2 - 9} + \frac{x}{x^2 + x - 12} \geq \frac{3x}{x^2 + 7x + 12}$$

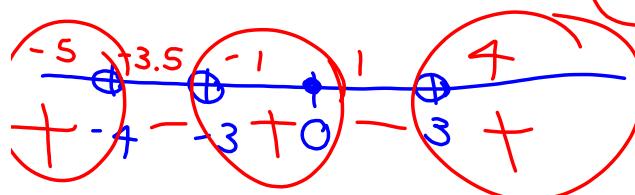
$$\frac{\cancel{2x}}{(x-3)(x+3)} + \frac{x}{(x+4)(x-3)} - \frac{\cancel{3x}}{(x+4)(x+3)} \geq 0$$

$$\frac{2x(x+1) + x(x+3) - 3x(x-3)}{(x-3)(x+3)(x+4)} \geq 0$$

$$\frac{2x^2 + 8x + x^2 + 3x - 3x^2 + 9x}{(x-3)(x+3)(x+4)} \geq 0$$

$$\frac{20x}{(x-3)(x+3)(x+4)} \geq 0$$

$(-\infty, -4) \cup (-3, 0]$
 $\cup (3, \infty)$



36. $x^5 + 24 > 3x^3 + 8x^2$

$$a^3 - b^3 =$$

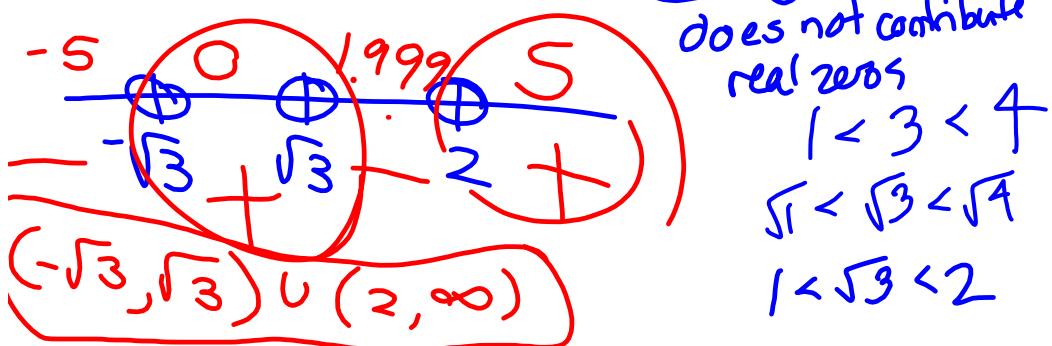
$$\underline{x^5 - 3x^3 - 8x^2 + 24 > 0}$$

$$(a-b)(a^2+ab+b^2)$$

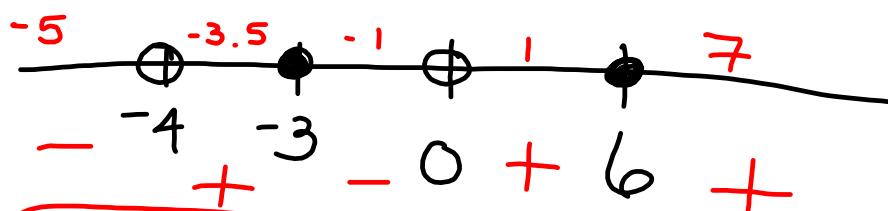
$$x^3(x^2 - 3) - 8(x^2 - 3) > 0$$

$$(x^2 - 3)(x^3 - 8) > 0$$

$$(x - \sqrt{3})(x + \sqrt{3})(x - 2)(x^2 + 2x + 4) > 0$$



$$\frac{(x-6)^2(x+3)}{x(x+4)} \leq 0$$



$$(-\infty, -4) \cup [-3, 0)$$

$$38. 2x^3 + x^2 < 10 + 11x$$

$$2x^3 + x^2 - 11x - 10 < 0$$

possible rational zeros: $\frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2}$

$$\begin{array}{r} -1 \\ \underline{-2} \quad 1 \quad -11 \quad -10 \\ \hline 2 \quad -1 \quad -10 \quad | \quad 0 \end{array}$$

$$(x+1)(2x^2 - x - 10) < 0$$

$$(x+1)(2x-5)(x+2) < 0$$

$$2(x - \frac{5}{2})(x+1)(x+2) < 0$$

$$\begin{array}{c} -3 \quad -1.5 \quad 0 \quad 3 \\ \hline - \quad + \quad + \quad - \\ -2 \quad + \quad - \quad \frac{5}{2} \quad + \\ (-\infty, -2) \cup (-1, \frac{5}{2}) \end{array}$$

Graph the rational function.

$$f(x) = \frac{x^2(x+6) - 1(x+6)}{x^3 + 6x^2 - 4x - 24} = \frac{(x+6)(x-2)(x+2)}{(x+3)(x-1)}$$

zeros: -6, 2, -2

V.A.: $x = -3, x = 1$

y-int: (0, 8)

end behavior:

$$\frac{x^3}{x^2} = x \quad y = x$$

