Review: Given the line $2x-3y=5$, determine the equations of both a line parallel to and perpendicular to this line which pass through the point $(1,1)$.

Point-Slope Equation: $y-y_1=m(x-x_1)$

$-3y=-2x+5$

$y=\frac{2}{3}x-\frac{5}{3}$

parallel

$m=2/3$

$y-1=\frac{2}{3}(x-1)$

$y=\frac{2}{3}x-\frac{2}{3}+1$

$y=2/3x+1/3$

perp: $m=-3/2$

$y-1=-\frac{3}{2}(x-1)$

$y=-\frac{3}{2}x+\frac{3}{2}+1$

$y=-\frac{3}{2}x+\frac{5}{2}$

3.7 Variation and Applications

2-variable direct variation -- "y varies directly with x"
$y=kx$, k=constant of variation

inverse/indirect variation -- "y varies inversely with x"
$y=k\cdot\frac{1}{x}$

8. Find the variation constant and write an equation of variation if y varies directly as x, and y=3 when x=33.

$y=k\cdot x$

$\frac{3}{33}=\frac{k\cdot 33}{33}$

$\frac{1}{11}=k$

$y=\frac{1}{11}x$

$y=\frac{x}{11}$

4. Find the variation constant and write an equation of variation if y varies inversely with x, and y=12 when x=5.

$y = k \cdot \frac{1}{x}$

$12 = k \cdot \frac{1}{5}$

$60 = k$

$y = 60 \cdot \frac{1}{x}$

$y = \frac{60}{x}$

34. Find the variation constant and write an equation of variation if y varies jointly as x and z and inversely as the square of w. y=12/5 when x=16, z=3, and w=5.

*joint variation is direct variation with more than one variable

$y = k \cdot \frac{xz}{w^2}$

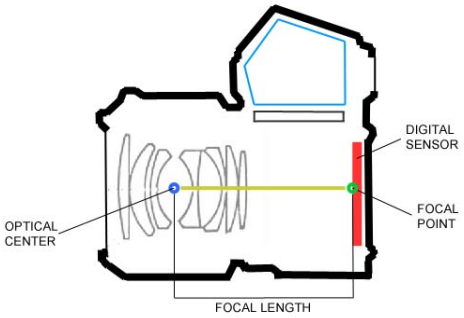
$\frac{12}{5} = k \cdot \frac{16 \cdot 3}{5^2}$

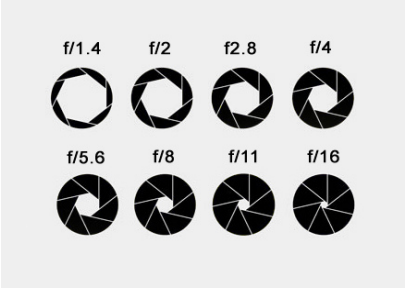
$\frac{5^2}{16 \cdot 3} \cdot \frac{12^4}{5} = k$

$k = \frac{5}{4}$

$y = \frac{5}{4} \cdot \frac{xz}{w^2}$

$y = \frac{5xz}{4w^2}$





22. The f-stop of a ~~23.5 mm diameter~~ lens is directly proportional to the focal length F of the lens. If a 150-mm focal ~~has~~ an f-stop of 6.3, find the f-stop of ~~a 23.5 mm diameter~~ lens with a focal length of 80 mm.

$f = k \cdot F$

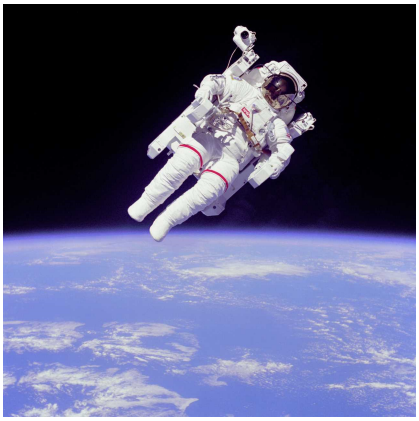
$6.3 = k \cdot 150$

$\frac{6.3}{150} = k$

$f = \frac{6.3}{150} \cdot 80$

$= \frac{6.3 \cdot 8}{15}$

≈ 3.36



38. The weight W of an object varies inversely as the square of the distance d from the center of the earth. At sea level (3978 mi from the center of the earth), an astronaut weighs 220 lb. Find his weight when he is 200 mi above the surface of the earth.

$$W = k \cdot \frac{1}{d^2} = \frac{k}{d^2}$$

$$220 = k \cdot \frac{1}{3978^2}$$

$$k = 220(3978)^2$$

$$W = \frac{200(3978)^2}{(3978+200)^2}$$

$$= \frac{200(3978)^2}{4178^2}$$

$$\approx 181.3 \text{ lb}$$

4.1 Inverse Functions

Recall:

f is a function if each input value (x) has exactly one output $f(x)$

Functions pass the vertical line test.

f is a one-to-one function if, in addition, each y corresponds to only one x.

One-to-one functions pass both the horizontal line test and the vertical line test.

Formally, a function is one-to-one if different inputs have ~~the same~~ different outputs, i.e.

$if\ a \neq b, then\ f(a) \neq f(b),$

Or equivalently, f is one-to-one if when the outputs are the same, the inputs are the same, i.e.

$if\ f(a) = f(b), then\ a = b.$

Proving that a function *is* one-to-one v. proving that a function *is not* one-to-one
(problems 17-24 from section 4.1)

To show that f(x) is not one-to-one, it is enough to provide a single counter-example, i.e. 2 different inputs that yield the same output

$f(x) = x^2 - 5$

$f(1) = (1)^2 - 5 = 1 - 5 = -4$

$f(-1) = (-1)^2 - 5 = 1 - 5 = -4$

Since $f(1) = f(-1)$, f is not 1-1

To show that f(x) is one-to-one, we must prove it in general.

$f(x) = -2x^3 + 1$

$f(a) = f(b)$

$-2a^3 + 1 = -2b^3 + 1$

$-1 \quad -1$

$\frac{-2a^3}{-2} = \frac{-2b^3}{-2}$

$\sqrt[3]{a^3} = \sqrt[3]{b^3}$

$a = b$

Since $f(a) = f(b)$ implies that $a = b$, f is one-to-one!

Homework #7

3.7	#23-37 odd	variation and applications
4.1	#17-23 odd	prove f is one-to-one; prove g is not one-to-one
	#59-63 odd	determine if f is on-to-one and if so, determine its inverse
	#77-81 odd	sketch the inverse function by reflecting over y=x
	#83-87 odd	use composition to show that the functions are inverses