

Application: Compound Interest

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

A = amount of money

P = principal (initial investment)

t = time in # of years

r = interest rate (decimal)

n = # of times interest is compounded per year

Example:

\$100 @ 5% interest compounded quarterly for 1 year

$$A = 100 \left(1 + \frac{0.05}{4}\right)^4$$

$$\boxed{\$105.09}$$

If we invest \$1 at 100% interest for 1 year,

$$A = P \left(1 + \frac{r}{n}\right)^{nt} \quad P = 1, r = 1, t = 1$$

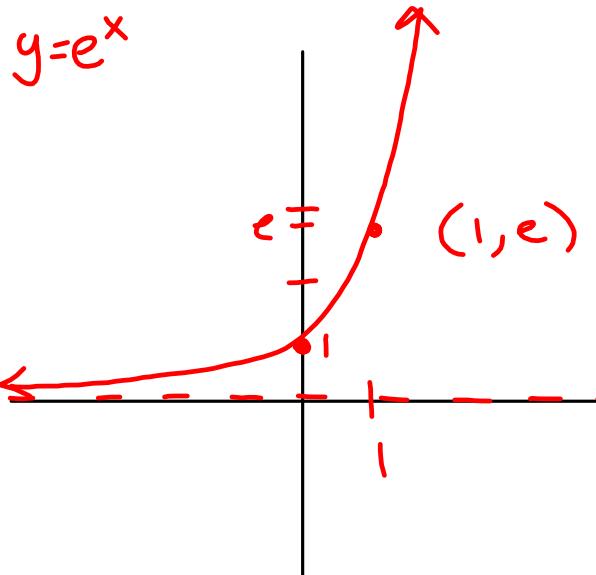
n	$A(n) = \left(1 + \frac{1}{n}\right)^n$
1 annually	$\left(1 + \frac{1}{1}\right)^1 = 2$
2 semiannually	$\left(1 + \frac{1}{2}\right)^2 = 2.25$
4 quarterly	$\left(1 + \frac{1}{4}\right)^4 = 2.44$
12 monthly	$\left(1 + \frac{1}{12}\right)^{12} = 2.61$
365 daily	$\left(1 + \frac{1}{365}\right)^{365} = 2.7145$
8760 hourly	$\left(1 + \frac{1}{8760}\right)^{8760} = 2.7181$
--> ∞	e

$$e \approx 2.718$$

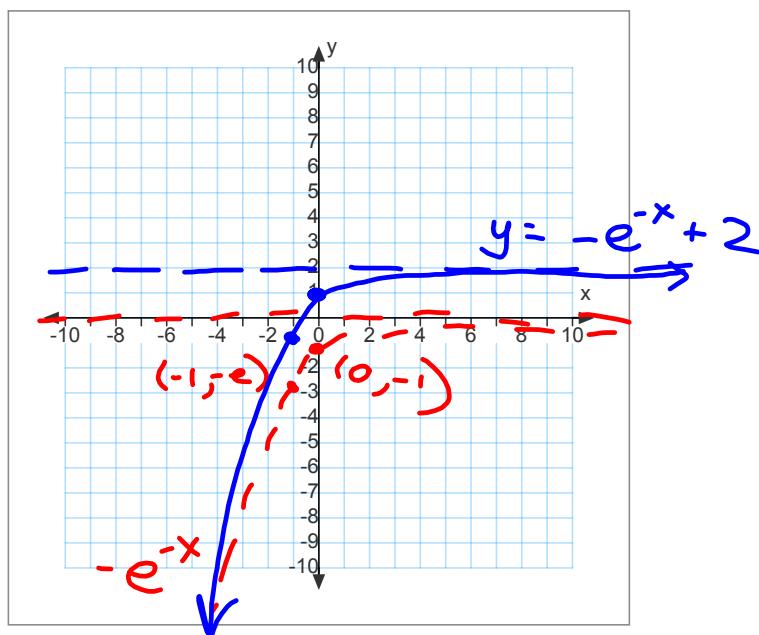
$$e^2 \approx e^2 \approx 7.389$$

$$e^{-0.23} \approx 0.7945$$

$$e^{-0.23}$$



$$y = -e^{-x} + 2$$



4.3 Logarithmic Functions

Inverses of Exponential Functions

$$f(x) = 2^x$$

$$y = 2^x$$

$$x = 2^y$$

y = the power to which we raise 2

$$f^{-1}(x) = \text{_____} \text{ to get } x$$

$$f^{-1}(x) = \log_2 x \quad \text{"log, base 2, of } x\text{"}$$

$$\log_2 8 = 3 \iff 2^3 = 8$$

\uparrow
3 is the power to which
we raise 2 to get 8

Simplify/evaluate :

$$\log_{10} 1000 = 3 \quad \longleftrightarrow \quad 10^3 = 10 \cdot 10 \cdot 10 = 1000$$

$$\log_{10} 0.001 = -3 \quad \longleftrightarrow \quad 10^{-3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1000}$$

$$\log_3 27 = 3$$

$$\log_5 1 = \textcircled{0}$$

$$\log_6 6 = 1$$

* $\log_a 1 = 0$
for all valid a

$$\log_a a = 1$$

For $f(x) = a^x$, the inverse function is

$$f^{-1}(x) = \log_a x.$$

$y = \log_a x$ is the number y such that $x = a^y$, where $x > 0$ and a is a positive constant other than 1.

log graphs :

Vertical asymptote

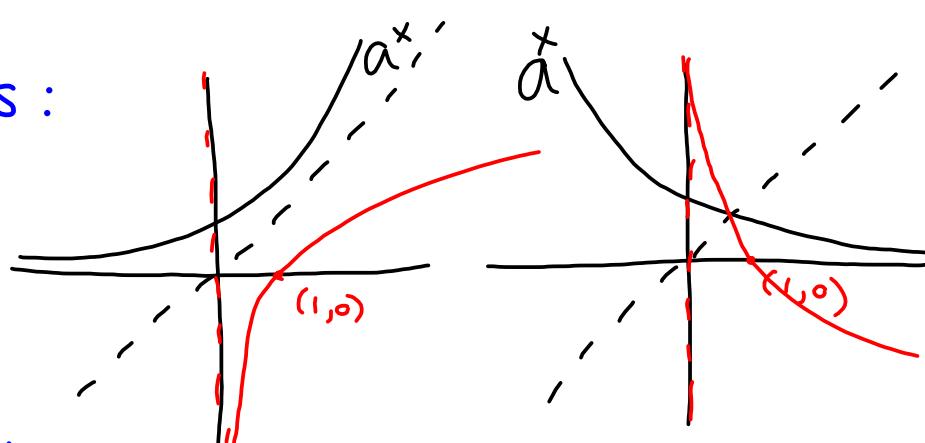
$$\bullet x = 0$$

$$x\text{-int: } (1, 0)$$

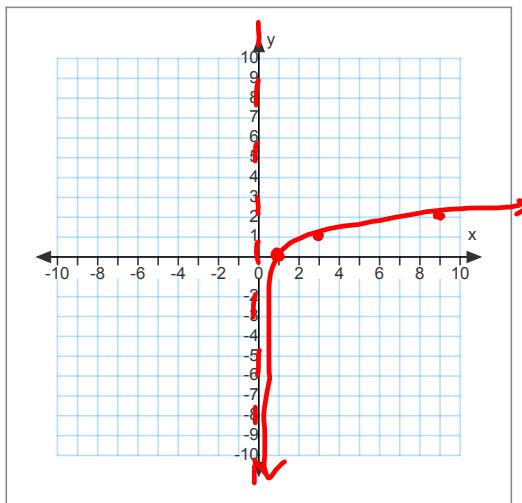
$$\text{domain: } (0, \infty)$$

$$\text{range: } (-\infty, \infty)$$

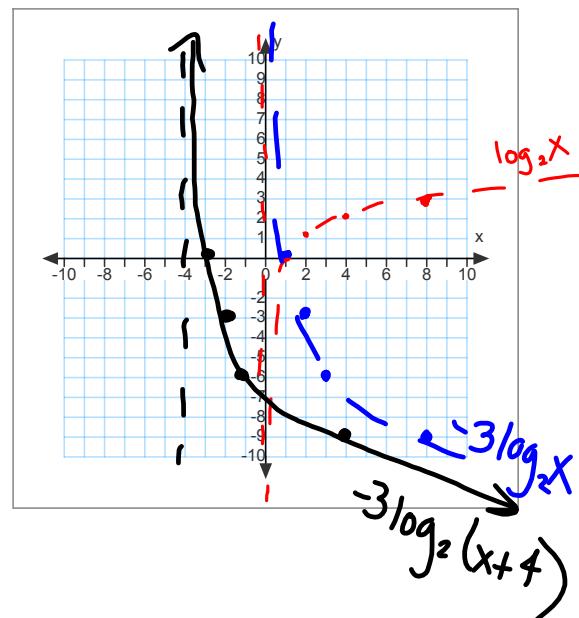
$$a > 1$$



$$y = \log_3 x$$



$$y = -3 \log_2(x+4)$$



$$\log_a 1 = 0$$

&

$$\log_a a = 1$$

for any base a

$$\log_a x = y \leftrightarrow a^y = x$$

$$32 = 2^x \leftrightarrow \log_2 32 = x \rightarrow x = 5$$

$$\log_2 64 = x \leftrightarrow 64 = 2^x \rightarrow x = 6$$

Log on your calculator

is $\log_{10} x = \log x$
"common log"

ln is $\log_e x = \ln x$
"natural log"

Change of Base Formula

$$\log_b M = \frac{\log_a M}{\log_a b} = \frac{\ln M}{\ln b} = \frac{\ln M}{\ln b}$$

$$\log_6 7 = \frac{\log 7}{\log 6} = \frac{\ln 7}{\ln 6}$$

4.3-4.4 Logarithmic Functions, Graphs, & Properties

$$\log_2 64 = 6 \quad \longleftrightarrow \quad 2^6 = 64$$

$$\log_{81} 3 = \frac{1}{4} \quad \longleftrightarrow \quad 81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$

$$\ln e = \log_e e = 1$$

$$\ln e^2 = \log_e e^2 = 2$$

$$\ln 1 = 0$$

$$\ln \sqrt[3]{e} = \log_e e^{\frac{1}{3}} = \frac{1}{3}$$

$$\log_a \hat{a}^x = x$$

Homework #7 (due Fri. 9/26)

- 3.7 #23-37 odd variation and applications
- 4.1 #17-23 odd prove f is one-to-one; prove g is not one-to-one
 #59-63 odd determine if f is on-to-one and if so, determine its inverse
 #77-81 odd sketch the inverse function by reflecting over y=x
 #83-87 odd use composition to show that the functions are inverses
- 4.2 #5-10 all match an exponential function to its graph
 #11-41 odd sketch graphs of exponential functions using transformations
 #43a,b,c,45,47 compound interest word problems
- 4.3 #1-8 all sketch graphs of logarithmic functions
 #9-33 odd evaluate log expressions without a calculator ←
 #35-53 odd convert between logarithmic and exponential expressions
 #69-77 odd apply change of base formula & calculator to approximate log expressions
 #83-90 all graph logarithmic functions using transformations