Turn in Homework #7

3.7 #23-37 odd variation and applications

4.1 #17-23 odd prove f is one-to-one; prove g is not one-to-one
#59-63 odd determine if f is on-to-one and if so, determine its inverse
#77-81 odd sketch the inverse function by reflecting over y=x
#83-87 odd use composition to show that the functions are inverses

4.2 #5-10 all match an exponential function to its graph
#11-41 odd sketch graphs of exponential functions using transformations
#43 a, b, c, 45,47 compound interest word problems

Review: **Inverse Functions**

A function is one-to-one if $f(a) = f(b)$ implies that $a = b$ for all a, b in the domain of f. That is, in addition to being a function (each x maps to exactly one y), a one-to-one function only has one x-value mapping to each y-value. The graph of a one-to-one function passes both the horizontal and vertical line tests.

Example of verifying that a function is one-to-one:

$f(x) = (x+4)^3 - 5$
$f(a) = f(b)$
$(a+4)^3 - 5 = (b+4)^3 - 5$
$(a+4)^3 = (b+4)^3$
$\sqrt[3]{(a+4)^3} = \sqrt[3]{(b+4)^3}$
$(a + 4) = (b + 4)$
$a = b$
Since $f(a) = f(b)$ implies that $a = b$, f is one-to-one.

Example showing that a function is NOT one-to-one:

$f(x) = x^2$
$f(2) = 4$
$f(-2) = 4$
Since two different x-values yield the same y-value, f is not one-to-one.

The one-to-one functions $f(x)$ and $g(x)$ are inverses if:
$(f \circ g)(x) = x$ and $(g \circ f)(x) = x$

Example of verifying that two functions are inverses:

$f(x) = x^3 \quad g(x) = \sqrt[3]{x}$
$(f \circ g)(x) = \left(\sqrt[3]{x}\right)^3 = x$
$(g \circ f)(x) = \sqrt[3]{(x^3)} = x$

How to find an inverse function:

1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for y in terms of x.
4. Replace y with $f^{-1}(x)$.

Example of finding an inverse function:

$$f(x) = \frac{5x-3}{2x+1}$$
$$y = \frac{5x-3}{2x+1}$$
$$x = \frac{5y-3}{2y+1}$$
$$x(2y+1) = 5y-3$$
$$2xy + x = 5y - 3$$
$$x + 3 = 5y - 2xy$$
$$x + 3 = y(5-2x)$$
$$y = \frac{3+x}{5-2x}$$
$$\boxed{f^{-1}(x) = \frac{3+x}{5-2x}}$$

Properties of Exponential Functions:

$a^m a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

$(a^m)^n = a^{mn}$

$(a^p b^q)^r = a^{pr} b^{qr}$, $\left(\frac{a^p}{b^q}\right)^r = \frac{a^{pr}}{b^{qr}}$

$a^{-1} = \frac{1}{a}$

$a^0 = 1, a \neq 0$

$\sqrt[n]{a} = a^{1/n}$

Properties of Logarithmic Functions:

Product Rule: $log_a(MN) = log_a M + log_a N$

Power Rule: $log_a(M)^P = p\, log_a M$

Quotient Rule: $log_a\left(\frac{M}{N}\right) = log_a M - log_a N$

Change of Base Formula: $log_b M = \frac{log_a M}{log_a b}$

Other Properties: $log_a a = 1$ $\quad log_a 1 = 0$

$log_a a^x = x$ $\quad a^{log_a x} = x$

$\ln x = \log_e x$ $\quad \log x = \log_{10} x$

The logarithmic equation $\log_a b = c$ is equivalent to the exponential equation $a^c = b$

Compound Interest

The amount of money A that a principal P will grow to after t years at interest rate r (in decimal form), compounded times per year, is given by the formula

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$e \approx 2.7$

$\log_a 2 \approx 0.301, \log_a 7 \approx 0.845, \log_a 11 \approx 1.041$

54. $\log_a 14 = \log_a(7 \cdot 2) = \log_a 7 + \log_a 2$

$\approx 0.845 + 0.301 = 1.146$ $\quad = \boxed{1.146}$

56. $\log_a \frac{1}{7} = \log_a 7^{-1} = (-1)\log_a 7$

$\approx \boxed{-0.845}$

or $= \log_a 1 - \log_a 7$

$= 0 - \log_a 7$ //

58. $\log_a 9$ ~~$= \log_a(7+2)$~~

~~$\log_a 7 + \log_a 2 = \log_a 14$~~ $\neq \log_a 9$

not possible w/ given information

Express in terms of sums and differences of logarithms.

24. $\log_a x^3y^2z$ $= \log_a x^3 + \log_a y^2 + \log_a z$

$= \boxed{3\log_a x + 2\log_a y + \log_a z}$

26. $\log_b \frac{x^2y}{b^3}$ $= \log_b x^2 + \log_b y - \log_b b^3$

$= \boxed{2\log_b x + \log_b y - 3}$

Express in terms of sums and differences of logarithms.

30. $\ln \sqrt[3]{5x^5}$ $= \ln (5x^5)^{1/3}$ $= \ln(5^{1/3}x^{5/3})$

$= \ln 5^{1/3} + \ln x^{5/3}$ $= \boxed{\frac{1}{3}\ln 5 + \frac{5}{3}\ln x}$

34. $\log_a \sqrt{\frac{a^6b^8}{a^2b^5}}$ $= \log_a \sqrt{a^4b^3}$ $= \log_a (a^4b^3)^{1/2}$

$= \log_a (a^2 b^{3/2})$ $= \log_a a^2 + \log_a b^{3/2}$

$= \boxed{2 + \frac{3}{2}\log_a b}$

Express as a single logarithm and, if possible, simplify.

40. $\frac{1}{2}\log a - \log 2$ $= \log a^{1/2} - \log 2 =$

$= \log \frac{a^{1/2}}{2}$ $= \boxed{\log \frac{\sqrt{a}}{2}}$

48. $\log_a \frac{a}{\sqrt{x}} - \log_a \sqrt{ax}$ $= \log_a \left(\frac{\frac{a}{\sqrt{x}}}{\frac{\sqrt{ax}}{1}}\right)$

$= \log_a \left(\frac{a}{\sqrt{x}\sqrt{ax}}\right) = \log_a \left(\frac{a}{\sqrt{ax^2}}\right) = \log_a \frac{a}{x\sqrt{a}}$

$= \log_a \frac{a^1}{x a^{1/2}} = \log_a \frac{a^{1/2}}{x} = \boxed{\log_a \frac{\sqrt{a}}{x}}$

$\frac{a}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\cancel{a}\sqrt{a}}{\cancel{a}} = \sqrt{a}$

Express as a single logarithm and, if possible, simplify.

52. $120\left(\ln \sqrt[5]{x^3} + \ln \sqrt[3]{y^2} - \ln \sqrt[4]{16z^5}\right)$

$= 120 \ln \frac{\sqrt[5]{x^3} \cdot \sqrt[3]{y^2}}{\sqrt[4]{16z^5}}$

$= \ln \left(\frac{x^{3/5} y^{2/3}}{16^{1/4} z^{5/4}}\right)^{120} = \ln \frac{x^{\frac{3}{5}\cdot\frac{120}{1}} y^{\frac{2}{3}\cdot\frac{120}{1}}}{16^{\frac{1}{4}\cdot\frac{120}{1}} z^{\frac{5}{4}\cdot\frac{120}{1}}}$

$= \boxed{\ln \frac{x^{72} y^{80}}{2^{120} z^{150}}}$

Simplify.

66. $\log_t t^{2713}$ $= 2713$

68. $\log_q q^{\sqrt{3}}$ $= \sqrt{3}$

70. $5^{\log_5(4x-3)}$ $= 4x-3$

72. $e^{\ln x^3}$ $= x^3$

74. $\log 10^{-k}$ $= \log_{10} 10^{-k} = \boxed{-k}$

76. $\log_b \sqrt{b^3}$ $= \log_b b^{3/2} = \boxed{3/2}$

$$\boxed{a^{\log_a x} = x}$$

$$\boxed{\log_a (a^x) = x}$$

Determine whether the statement is true. Assume that a, x, M, and N are positive.

102. $\log_N (MN)^x = x \log_N M + x$

$LHS = \log_N (MN)^x = x \log_N (MN) =$

$= x [\log_N M + \log_N N] =$

$= x \log_N M + x \log_N N = x \log_N M + x = RHS$

True!

Write without using logarithms.

106. $\log_a x + \log_a y - mz = 0$

$\log_a b = c \Leftrightarrow a^c = b$

$\log_a(xy) = mz$

$\boxed{a^{mz} = xy}$

HW #8 (due Fri. 10/3)

4.3	#1-8 all	sketch graphs of logarithmic functions
	#9-33 odd	evaluate log expressions without a calculator
	#35-53 odd	convert between logarithmic and exponential expressions
	#69-77 odd	apply change of base formula & calculator to approximate log expressions
	#83-90 all	graph logarithmic functions using transformations
4.4	# 31,33, 49-55 odd; 65-75 odd; 107	applying log rules
4.5	# 1-25 odd;	solving exponential equations
	#27-47	odd solving logarithmic equations
4.6	#5,7,9,15,17	application problems

Quiz #3 - Monday 9/29
Test #3 - Wed 10/8