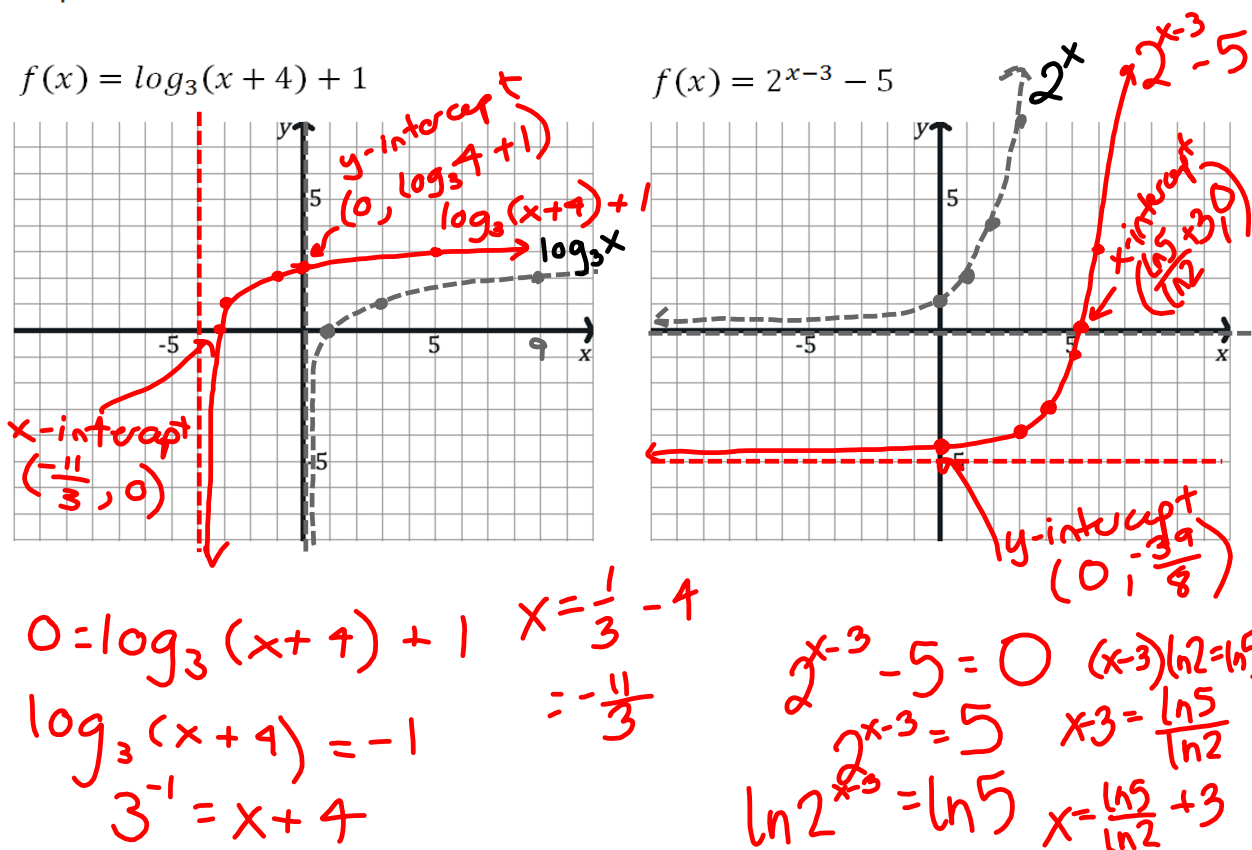


Graph



4.6 Applications & Models

$$P(t) = P_0 e^{kt}, \quad k > 0$$

Population Growth

$P(t)$ = population at time t

P_0 = initial population

k = exponential growth rate

Continuously Compounded Interest

$P(t)$ = amount of \$ at time t

P_0 = initial investment

k = interest rate

$$P_0 = P(0)$$

How to find Growth Rate / Doubling Time:

$$P(t) = P_0 e^{kt}$$

We need to divide by P_0 in order to isolate the exponential expression before taking a log to get the variable out of the exponent.

$$\frac{P(t)}{P_0} = e^{kt}$$

$$\ln\left(\frac{P(t)}{P_0}\right) = kt$$

The **doubling time** T is the time it takes for the population to double. Note that this is independent of the initial population.

$$P(T) = 2P_0$$

$$2P_0 = P_0 e^{kT}$$

$$2 = e^{kT}$$

$$\ln 2 = kT$$

$$T = \frac{\ln 2}{k}$$

2. exponential growth rate of rabbits is 11.7% per day;
initial population of 100 rabbits

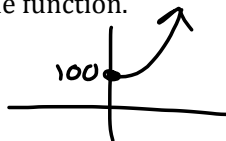
$$P_0 = 100; k = 0.117$$

$$P(t) = P_0 e^{kt}$$

- a. Find the exponential growth function.

$$P(t) = 100e^{0.117t}$$

- b. Graph the function.



- c. What will the population be after 7 days?

$$P(7) = 100e^{0.117 \cdot 7} = 226 \text{ rabbits}$$

- d. Find the doubling time.

$$T = \frac{\ln 2}{0.117} = 5.9 \text{ days}$$

OR

$$\frac{200}{100} = \frac{100 e^{0.117 \cdot T}}{100}$$

$$2 = e^{0.117T}$$

$$\ln 2 = \ln e^{0.117T}$$

$$\ln 2 = 0.117T$$

$$\frac{\ln 2}{0.117} = T$$

8. Interest Compounded Continuously $P(t) = P_0 e^{kt}$

Initial Investment $P_0 = \frac{P(5)}{e^{k \cdot 5}}$	Interest Rate $k = \frac{\ln(\frac{P(5)}{P_0})}{5}$	Doubling Time $T = \frac{\ln 2}{k}$	Amount after 5 years $P(5) = P_0 e^{k \cdot 5}$
a. \$35,000	6.2%	$\frac{\ln 2}{0.062} = 11.2 \text{ yrs}$	$35000 e^{0.062(5)} = \$47,719.88$
b. \$5,000	$\frac{\ln(\frac{7130.90}{5000})}{5} = 7.12$	$\frac{\ln 2}{0.071} = 9.76 \text{ yrs}$	\$7,130.90
c. $\frac{11414.71}{e^{0.084(5)}} = \$7,499.99$	8.4%	$\frac{\ln 2}{0.084} = 8.3 \text{ yrs}$	\$11,414.71
d. $\frac{17539.32}{e^{(0.063)(5)}} = \$12,800$	$\frac{\ln 2}{11} = 6.32$	11 years	\$17,539.32

10. carbon dating

a statue has lost 35% of its carbon-14; how old is it?

* the half-life of carbon-14 is 5750 years

if it lost 35% it still has 65%

$$P(t) = 0.65 P_0$$

↑ the amount of time it takes for the amount to be cut in half

$$P(T) = \frac{1}{2} P_0$$

$$\frac{1}{2} P_0 = P_0 e^{k \cdot T}$$

$$\frac{1}{2} = e^{k \cdot T}$$

$$\ln \frac{1}{2} = k \cdot T$$

$$\frac{\ln \frac{1}{2}}{k} = T \quad \text{or} \quad \frac{\ln \frac{1}{2}}{T} = k$$

↑ half-life ↑ decay rate

$$k = \frac{\ln \frac{1}{2}}{5750} \approx -0.000120547$$

$$0.65 P_0 = P_0 e^{k \cdot t}$$

$$\ln 0.65 = k \cdot t$$

$$t = \frac{\ln 0.65}{-0.000120547} = 3433 \text{ yrs}$$

HW #8 (due Fri. 10/3)

4.3 #1-8 all sketch graphs of logarithmic functions #9-33 odd evaluate log expressions without a calculator

#35-53 odd convert between logarithmic and exponential expressions #69-77 odd apply change of base formula & calculator to approximate log expressions #83-90 all graph logarithmic functions using transformations

4.4 # 31,33, 49-55 odd; 65-75 odd; 107 applying log rules

4.5 # 1-25 odd; solving exponential equations
#27-47 odd solving logarithmic equations

Due Mon 10/6

4.6 #5,7,9,15,17 application problems

Test #3 - Wed 10/8