b 1. $\log_a(MN) =$

a. $\log_a M \log_a N$ b. $\log_a M + \log_a N$ c. $\log_a(M+N)$ d. $N \log_a M$

c 2. $\log_a 1 =$

a. a b. 1 c. 0 d. e^a

d 3. $\log_a\left(\frac{M}{N}\right) =$

a. $\frac{\log_a M}{\log_a N}$ b. $\frac{\ln M}{\ln a}$ c. $\log_a(M-N)$ d. $\log_a M - \log_a N$

b 4. $\log_a a =$

a. a b. 1 c. 0 d. e^a

c 5. $\log x =$

a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

a 6. $\ln x =$

a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

b 7. $\log_a M^p =$

a. $(\log_a M)^p$ b. $P \log_a M$ c. $(\log a)(M^p)$ d. $MP \log a$

b 8. $\log_a b =$

a. $\frac{\log_b x}{\log_a x}$ b. $\frac{\log b}{\log a}$ c. $b \log a$ d. $\frac{\log a}{\log b}$

Limited Population Growth

Logistic Function

$$P(t) = \frac{a}{1+be^{-kt}}$$

$e^{-kt} = \frac{1}{e^{kt}}$

as $t \to \infty$, $e^{kt} \to \infty$

$\frac{1}{e^{kt}} \to 0$ & $P(t) \to a$

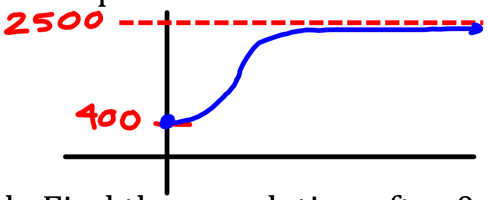
$\Rightarrow$ horizontal asymptote

$y = a$

16. limited population growth in a lake
P_o=400 fish
limiting value is 2500

$$P(t)=\frac{2500}{1+5.25e^{-0.32t}} \quad , \quad t \text{ in months}$$

a. Graph the function.



b. Find the population after 0, 1, 5, 10, 15, & 20 months.

$$P(15)=\frac{2500}{1+5.25e^{(-0.32)(15)}} = \boxed{2396 \text{ fish}}$$

20. The police discover the body of a murder victim. Critical to solving the crime is determining when the murder was committed. The coroner arrives at the murder scene at 12:00 pm. She immediately takes the temperature of the body and finds it to be 94.6°. She then takes the temperature 1 hour later and finds it to be 93.4°. The tempurature of the room is 70°. **When was the murder committed?**

$P(t)=P_0e^{kt}$

time 0 = 12:00 pm

P = actual body temperature − room temperature
(degrees above room temperature)

$P(0)=P_0 = 94.6^\circ - 70^\circ = 24.6^\circ$

$P(1) = 93.4^\circ - 70^\circ = 23.4^\circ$

we want to find t when $P(t) = 98.6^\circ - 70^\circ = 28.6^\circ$

$$\frac{23.4^\circ}{24.6^\circ} = \frac{24.6^\circ e^{k\cdot 1}}{24.6^\circ}$$

$$\frac{23.4}{24.6} = e^{k}$$

$$\ln\frac{23.4}{24.6} = k$$

$$P(t) = 24.6e^{\ln\frac{23.4}{24.6}\cdot t}$$

$$28.6 = 24.6e^{\ln\frac{23.4}{24.6} t}$$

$$\frac{28.6}{24.6} = e^{\ln\frac{23.4}{24.6} t}$$

$$\ln\frac{28.6}{24.6} = \ln\frac{23.4}{24.6} t$$

$$-3 \approx \frac{\ln\frac{28.6}{24.6}}{\ln\frac{23.4}{24.6}} = t$$

$\boxed{\text{The murder was committed @ about 9:00 am}}$

18. A certain element has a half-life of 30 years. If we start with a 5000 gram sample of the element, after how many years will there be only 100 grams left?

a. Find the exponential decay constant, k. Leave the answer in terms of the natural log.

$P(0) = 5000$

$P(30) = 2500$

$P(t) = P_0 e^{kt}$

$2500 = 5000e^{k \cdot 30}$

$\frac{1}{2} = e^{k \cdot 30}$

$\ln\frac{1}{2} = k \cdot 30$

$\boxed{k = \frac{\ln\frac{1}{2}}{30}}$

b. State the exponential decay function for the amount of substance left after time t, with correct values for P_0 and k.

$P(t) = 5000e^{\frac{\ln 1/2}{30}t}$

c. Determine the number of years t it will take for there to be only 100 grams of the element left.

want t when $P(t) = 100$

$100 = 5000e^{\frac{\ln 1/2}{30}t}$

$\frac{100}{5000} = e^{\frac{\ln 1/2}{30}t}$

$\ln\frac{100}{5000} = \frac{\ln 1/2}{30}t$

$t = \frac{\ln\frac{100}{5000}}{\frac{\ln 1/2}{30}}$

≈ 169.3 yrs

HW #8 (due Fri. 10/3)

4.3 #1-8 all sketch graphs of logarithmic functions
#9-33 odd evaluate log expressions without a calculator
#35-53 odd convert between logarithmic and exponential expressions
#69-77 odd apply change of base formula & calculator to approximate log expressions
#83-90 all graph logarithmic functions using transformations

4.4 # 31,33, 49-55 odd; 65-75 odd; 107 applying log rules

4.5 # 1-25 odd solving exponential equations
#27-47 odd solving logarithmic equations

Due Mon 10/6
4.6 #5,7,9,15,17 application problems

Test #3 - Wed 10/8