Evaluate the following without using a calculator:

$\log_4 \sqrt[3]{4} = 1/3$

$\ln 1 = 0$

$\log \sqrt[5]{10} = 1/5$

$\ln e = 1$

$\log_4 8 = \dfrac{\log_2 8}{\log_2 4} = \boxed{\dfrac{3}{2}}$

$10^{3\log 2} = 10^{\log 2^3} = 10^{\log 8} = 8$

$\log_{27} 3 = \dfrac{1}{3}$

$\log_5(1/125) = -3$

Express in terms of sums and differences of logarithms:

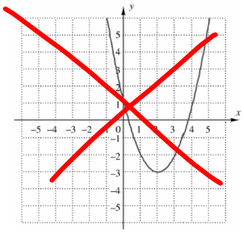
$\log \dfrac{\sqrt{x^3 y^5}}{z^2} = \log \dfrac{(x^3y^5)^{1/2}}{z^2} = \log \dfrac{x^{3/2}y^{5/2}}{z^2}$

$= \log x^{3/2} + \log y^{5/2} - \log z^2$

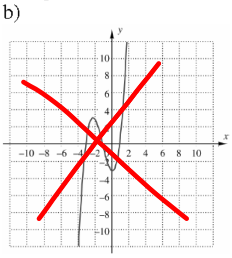
$= \boxed{\frac{3}{2}\log x + \frac{5}{2}\log y - 2\log z}$

2. Determine which of the following functions is one-to-one.

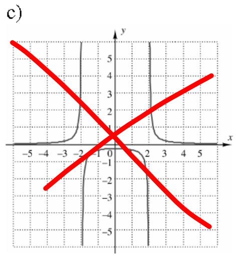
a)



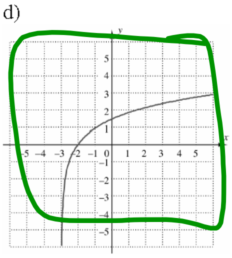
b)



c)



d)



Given that $\log_a 3 = 0.5000$ and $\log_a 2 = 0.3155$, find $\log_a 12$.

$\log_a 12 = \log_a(4\cdot 3) = \log_a(2^2\cdot 3) = \log_a 2^2 + \log_a 3$

$= 2\log_a 2 + \log_a 3 = \boxed{2(0.3155) + 0.5}$

Given that $\log_a 3 = 0.6131$ and $\log_a 12 = 1.3869$, find $\log_a 9$.

$\log_a 9 = \log_a 3^2 = 2\log_a 3 = \boxed{2(0.6131)}$

. Find the inverse function, $f^{-1}(x)$.

$f(x) = \dfrac{1}{x-3}$

$y = \dfrac{1}{x-3}$

$x = \dfrac{1}{y-3}$

$x(y-3) = 1$

$y - 3 = \dfrac{1}{x}$

$y = \dfrac{1}{x} + 3$

$\boxed{f^{-1}(x) = \dfrac{1}{x} + 3}$

Prove that the function $f(x) = 3(x-5)^2 + 7$ is <u>not</u> one-to-one.

$f(6) = 3(6-5)^2 + 7 = 3(1)^2 + 7 = 10$

$f(4) = 3(4-5)^2 + 7 = 3(-1)^2 + 7 = 10$

$4 \neq 6$, but $f(4) = f(6)$,
therefore f is <u>not</u> one-to-one

1. Determine whether the function is one-to-one, and if it is one-to-one, find a formula for its inverse.

a. $f(x) = 2x - 1$

$f(a) = f(b)$
$2a - 1 = 2b - 1$
$2a = 2b$
$a = b$

because $f(a) = f(b)$ implies $a = b$,
f is one-to-one

$y = 2x - 1$
$x = 2y - 1$
$x + 1 = 2y$
$y = \frac{x+1}{2}$

$$\boxed{f^{-1}(x) = \frac{x+1}{2}}$$

b. $f(x) = \frac{4}{x+7}$

$f(a) = f(b)$
$\frac{4}{a+7} = \frac{4}{b+7}$
$4(b+7) = 4(a+7)$
$b + 7 = a + 7$
$b = a$

because $f(a) = f(b)$ implies that $a = b$, f <u>is</u> one-to-one

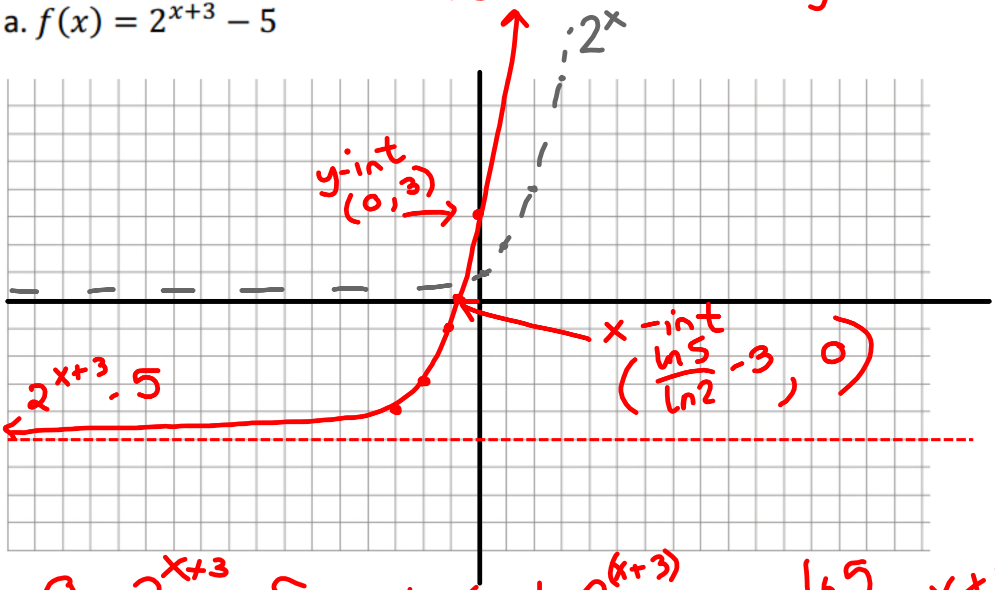
$y = \frac{4}{x+7}$
$x = \frac{4}{y+7}$
$x(y+7) = 4$
$y + 7 = \frac{4}{x}$
$y = \frac{4}{x} - 7$

$$\boxed{f^{-1}(x) = \frac{4}{x} - 7}$$

2. Graph the function. Include labels for asymptotes and at least two reference points.

determine x- & y-intercepts

a. $f(x) = 2^{x+3} - 5$



$0 = 2^{x+3} - 5$

$5 = 2^{x+3}$

$\ln 5 = \ln 2^{(x+3)}$

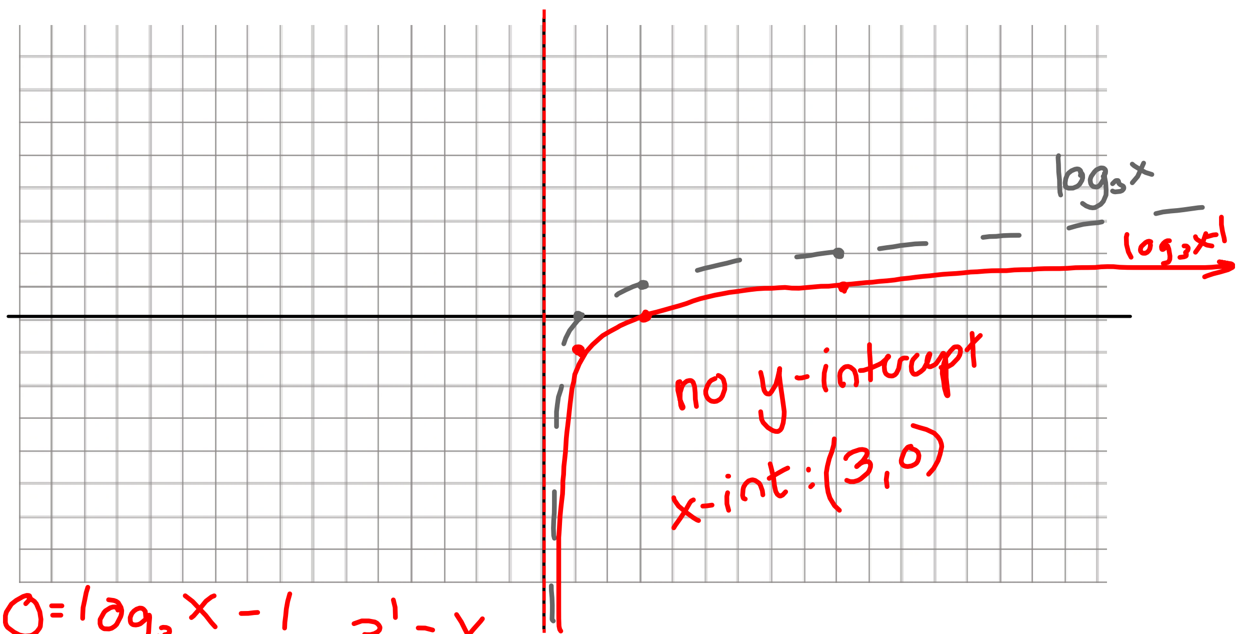
$\ln 5 = (x+3)\ln 2$

$\frac{\ln 5}{\ln 2} = x + 3$

$\frac{\ln 5}{\ln 2} - 3 = x$

& x- & y- intercepts

b. $f(x) = log_3 x - 1$



$0 = \log_3 x - 1$

$1 = \log_3 x$

$3^1 = x$

3. Find each of the following without using a calculator. Show the intermediate steps that led to the answer. Give exact answers.

a. $log_2 \frac{1}{4}$ $= -2$

b. $\log \sqrt{10}$ $= \frac{1}{2}$

c. $\ln \frac{1}{e^5}$ $= -5$

4. Find the logarithm using natural logs and the change-of-base formula. Give an exact answer in terms of logs ~~and an approximate answer to four decimal places.~~

a. $log_3 12$ $= \frac{\ln 12}{\ln 3}$

b. $log_{100} 15$ $= \frac{\ln 15}{\ln 100}$

5. Express as a single logarithm.

a. $\frac{2}{3}[\ln(x^2-9)-\ln(x+3)]+\ln(x+y)$

$= \frac{2}{3}\ln\frac{x^2-9}{x+3} + \ln(x+y)$

$= \frac{2}{3}\ln(x-3) + \ln(x+y)$

$= \ln(x-3)^{2/3} + \ln(x+y)$

$= \ln\left[(x-3)^{2/3}(x+y)\right]$

b. $\ln 2x + 3(\ln x - \ln y)$

$= \ln 2x + 3\ln\frac{x}{y} = \ln 2x + \ln\frac{x^3}{y^3} = \ln\left(2x\cdot\frac{x^3}{y^3}\right)$

$= \boxed{\ln\frac{2x^4}{y^3}}$

6. Express in terms of sums and differences of logs.

a. $\log\sqrt{x^3y}$

b. $\log_c\sqrt[3]{\frac{y^3z^2}{x^4}}$

Find the following in terms of

7.Given that $\log_b 2$ ~~≈ 0.693,~~ $\log_b 3$ ~~≈ 1.099,~~ *and* $\log_b 5$ ~~≈ 1.609, find the following to the nearest thousandth.~~

a. $\log_b \frac{1}{6} = \log_b 1 - \log_b (2 \cdot 3)$

$= 0 - (\log_b 2 + \log_b 3) = \boxed{-\log_b 2 - \log_b 3}$

b. $\log_b 30$

8. Simplify.

a. $5^{\log_5(4x-3)}$

b. $\log_b \sqrt{b^3}$

9. Solve for x. Find an exact answer algebraically.

a. $\log_2(x+1) + \log_2(x-1) = 3$

$\log_2 [(x+1)(x-1)] = 3$

$\log_2 (x^2-1) = 3$ $\qquad 2^{\log_2(x^2-1)} = 2^3$

$2^3 = x^2 - 1$

$8 = x^2 - 1$

$9 = x^2$

~~$x = \pm 3$~~ $\qquad \boxed{x = 3}$

b. $5^{4x-7} = 125$

$5^{4x-7} = 5^3$

$4x - 7 = 3$

$4x = 10$

$x = \frac{10}{4}$

$\boxed{x = \frac{5}{2}}$

10. In 1984, the average cell phone price was $3395, and in 2002, it was $145. Assuming the average price of a cell phone decreased according to the exponential model,

a. Find the value of k, and write an exponential function that describes the average price of a cell phone after time t, in years, where t is the number of years since 1984.

$K = \frac{\ln}{\ln}$ $P(t) = \boxed{P_0} e^{\boxed{K}t}$

b. Estimate the price of a cell phone in 2014 ~~2006 and 2008 to the nearest dollar~~.

c. At this decay rate, in what year will the price be $39?

HW #8 (due Fri. 10/3)

4.3 #1-8 all — sketch graphs of logarithmic functions
#9-33 odd — evaluate log expressions without a calculator
#35-53 odd — convert between logarithmic and exponential expressions
#69-77 odd — apply change of base formula & calculator to approximate log expressions
#83-90 all — graph logarithmic functions using transformations

4.4 # 31,33, 49-55 odd; 65-75 odd; 107 applying log rules

4.5 # 1-25 odd — solving exponential equations
#27-47 — odd solving logarithmic equations

Due Mon 10/6
4.6 #5,7,9,15,17 application problems

Test #3 - Wed 10/8

Upcoming: **Test #3**

Topics/problem types include:

- 4.1 - proving a function is or is not one-to-one; determining if two functions are indeed inverses; finding the inverse of a given function
- 4.2-4.3 - graphing log and exponential functions with transformations; evaluating log and exponential expressions without a calculator
- 4.4 - applying rules of logarithms and exponential functions to rewrite logarithmic and exponential expressions
- 4.5 - apply rules of logs and exponents, as well as properties of one-to-one functions to solve logarithmic and exponential equations
- 4.6 - write equations to describe and solve applied problems involving exponential growth and decay, compound interest, and limited growth