1. $\log_9\left(\frac{1}{3}\right) = \frac{\log_3 \frac{1}{3}}{\log_3 9} = \boxed{\frac{-1}{2}}$; $\log_9 \frac{1}{\sqrt{9}} = \log_9 \frac{1}{9^{1/2}} = \log_9 9^{-1/2} = \frac{-1}{2}$

2. $\log 1 = \boxed{0}$ $10^0 = 1$

3. $\log_8 32 = \frac{\log_2 32}{\log_2 8} = \boxed{\frac{5}{3}}$

4. $\log_{27} 3 = \boxed{\frac{1}{3}}$ $= \frac{\log_3 3}{\log_3 27} = \frac{1}{3}$

5. $\ln \sqrt[3]{e} = \ln e^{1/3} = \log_e e^{1/3} = \boxed{1/3}$

6. $2^{\log_2(6)} = \boxed{6}$

$\log_a a^x = x$

$a^{\log_a x} = x$

8. $\log_4(x+5) + \log_4(x-3) - \log_4(x) = 2$

$\log_4\left(\frac{(x+5)(x-3)}{x}\right) = 2$

$4^2 = \frac{(x+5)(x-3)}{x}$

$16 = \frac{x^2+2x-15}{x}$

$16x = x^2+2x-15$

$0 = x^2-14x-15$

$0 = (x-15)(x+1)$

$\boxed{x = 15}$; $\cancel{x=-1}$

9. $4^{5x+2} = \frac{1}{64}$

$\ln 4^{(5x+2)} = \ln \frac{1}{64}$

$(5x+2)\ln 4 = \ln \frac{1}{64}$

$5x+2 = \frac{\ln 1/64}{\ln 4}$

$5x = \frac{\ln 1/64}{\ln 4} - 2$

$x = \frac{\frac{\ln 1/64}{\ln 4} - 2}{5}$

$\frac{\log_4 \frac{1}{64} - 2}{5}$

$\frac{-3-2}{5} = -1$

$4^{5x+2} = 4^{-3}$

$5x+2 = -3$

$5x = -5$

$\boxed{x = -1}$

$4^{1/3} = \sqrt[3]{4} \neq \frac{1}{64}$

$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

$$P(t) = P_0 e^{kt}$$

$P(t)$ = amount at time t

P_0 = initial amount = $P(0)$

$e \approx 2.7$

k = growth/decay rate

t = time

Doubling time = T such that $P(T) = 2P_0$
(amt of time it takes the amount to double)

$$2P_0 = P_0 e^{k \cdot T}$$

Half-life = T such that $P(T) = \frac{1}{2}P_0$
(amt of time it takes the amount to be cut in half)

$$\frac{1}{2}P_0 = P_0 e^{k \cdot T}$$

Determine the <u>exponential growth constant</u> of a population whose doubling time is 5 years and initial population is 2,000,000 individuals.

$k = ?$

$P_0 = 2{,}000{,}000$

$5 = T$ such that $P(T) = 2P_0$

$P(5) = 4{,}000{,}000$

$$2\cancel{P_0} = \cancel{P_0} e^{k \cdot 5}$$

$$2 = e^{k \cdot 5}$$

$$\ln 2 = \ln e^{k \cdot 5}$$

$$\ln 2 = k \cdot 5$$

$$\boxed{k = \frac{\ln 2}{5}}$$

What was the <u>initial investment</u> into an account with a 6.5% interest rate if the amount in the account after 8 years is \$32,000?

$P_0 = ?$

$k = 0.065$

$P(8) = 32{,}000$

$$P(t) = P_0 e^{kt}$$

$$32000 = P_0 \cdot e^{(0.065)8}$$

$$\boxed{P_0 = \frac{32{,}000}{e^{0.065(8)}}}$$

What is the $\underbrace{\text{half-life}}$ in years of an element that $\underbrace{\text{has lost 90\% of its mass after 700 years}}$?

T such that
$P(T) = \frac{1}{2}P_0$

$\frac{\frac{1}{2}\cancel{P_0}}{\cancel{P_0}} = \frac{\cancel{P_0}e^{K\cdot T}}{\cancel{P_0}}$

$\frac{1}{2} = e^{K\cdot T}$

$\ln\frac{1}{2} = K\cdot T$

$T = \frac{\ln\frac{1}{2}}{K}$

$$\boxed{T = \frac{\ln\frac{1}{2}}{\left(\frac{\ln 0.1}{700}\right)} \text{ years}}$$

$P(700) = 0.1\cdot P_0$

$P(t) = P_0 e^{kt}$

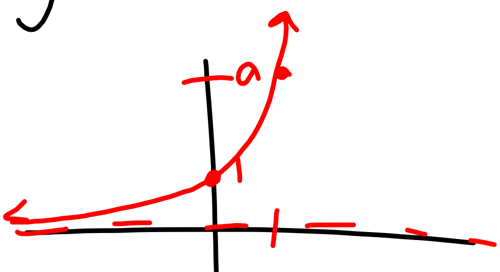
$0.1\cancel{P_0} = \cancel{P_0}e^{K\cdot 700}$

$0.1 = e^{K\cdot 700}$

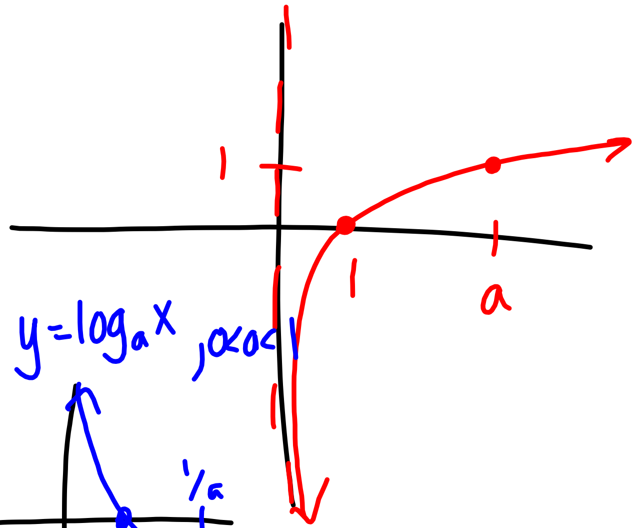
$\frac{\ln 0.1}{700} = \frac{K\cdot\cancel{700}}{\cancel{700}}$

$\frac{\ln 0.1}{700} = K$

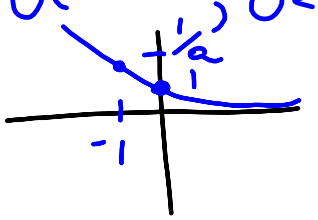
$y = a^x, a > 1$



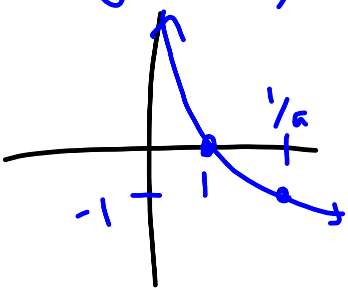
$y = \log_a x$



$y = a^x = \frac{1}{a^{-x}} = \left(\frac{1}{a}\right)^{-x}, 0 < a < 1$



$y = \log_a x, 0 < a < 1$



THURSDAY:

RE-TEST ON LOGS AND EXPONENTS

RECOMMENDED PRACTICE:

- SOLVING LOGARITHMIC AND EXPONENTIAL EQUATIONS (4.5 IN GREEN BOOK)
- SOLVING APPLICATION PROBLEMS (4.6 IN GREEN BOOK)