

$$\begin{cases} ax + by + cz = d \\ ax + by + cz = d \\ ax + by + cz = d \end{cases} \quad 30. \quad \begin{cases} 3x + 2y + 2z = 3 \\ x + 2y - z = 5 \\ 2x - 4y + z = 0 \end{cases}$$

$$\begin{array}{l} \left[\begin{array}{ccc|c} 3 & 2 & 2 & 3 \\ 1 & 2 & -1 & 5 \\ 2 & -4 & 1 & 0 \end{array} \right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 3 & 2 & 2 & 3 \\ 2 & -4 & 1 & 0 \end{array} \right] \\ \begin{aligned} 2 + (-3)(2) &= -4 \\ 2 + (-3)(-1) &= 5 \\ 3 + (-3)(5) &= -12 \end{aligned} \quad \begin{aligned} -4 + (-2)(2) &= -8 \\ 1 + (-2)(-1) &= 3 \\ 0 + (-2)(5) &= -10 \end{aligned} \\ \xrightarrow{\substack{R2 + (-3) \cdot R1 \\ R3 + (-2) \cdot R1}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -4 & 5 & -12 \\ 0 & -8 & 3 & -10 \end{array} \right] \xrightarrow{R2 \cdot \frac{1}{4}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & -\frac{5}{4} & -3 \\ 0 & -8 & 3 & -10 \end{array} \right] \\ \begin{aligned} -1 + (-2)(-\frac{5}{4}) &= -1 + \frac{5}{2} = -\frac{2}{2} + \frac{5}{2} = \frac{3}{2} \\ 5 + (-2)(3) &= -1 \end{aligned} \quad \begin{aligned} 3 + 8(-\frac{5}{4}) &= -7 \\ -10 + 8(3) &= 14 \end{aligned} \\ \xrightarrow{\substack{R1 + (-2) \cdot R2 \\ R3 + (8) \cdot R2}} \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & -1 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & 0 & -7 & 14 \end{array} \right] \xrightarrow{R3 \cdot \frac{-1}{7}} \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & -1 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & 0 & 1 & -2 \end{array} \right] \\ -1 + (-\frac{3}{2})(-2) = 2 \\ 3 + \frac{5}{4}(-2) = \frac{1}{2} \\ \xrightarrow{\substack{R1 + (-\frac{3}{2}) \cdot R3 \\ R2 + \frac{5}{4} \cdot R3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -2 \end{array} \right] \quad \boxed{\text{solution: } (2, \frac{1}{2}, -2)} \end{array}$$

$$\begin{cases} ax + by + cz = d \\ ax + by + cz = d \\ ax + by + cz = d \end{cases} \quad 34. \quad \begin{cases} x + y - 3z = 4 \\ 4x + 5y + z = 1 \\ 2x + 3y + 7z = -7 \end{cases}$$

$$\begin{array}{l} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 4 & 5 & 1 & 1 \\ 2 & 3 & 7 & -7 \end{array} \right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array} \right] \\ \begin{aligned} 5 + (-1)(1) &= 1 \\ 1 + (-1)(-3) &= 13 \\ 1 + (-1)(4) &= -15 \end{aligned} \quad \begin{aligned} 3 + (-2)(1) &= 1 \\ 7 + (-2)(-3) &= 13 \\ -7 + (-2)(4) &= -15 \end{aligned} \\ \xrightarrow{\substack{R2 + (-4) \cdot R1 \\ R3 + (-2) \cdot R1}} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 1 & 13 & -15 \end{array} \right] \xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

Row entirely of zeros

\Rightarrow infinitely many solutions

$$\begin{array}{l} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 13 & -15 \\ 0 & 0 & 0 & 0 \end{array} \right] \\ \begin{array}{l} \xrightarrow{\substack{R1 + (-1) \cdot R2 \\ R3 + (-1) \cdot R2}} \\ (f(y), y, g(y)) \\ (f(z), g(z), z) \end{array} \quad \begin{array}{l} \xrightarrow{R3 \cdot (-1)} \\ X + y - 3z = 4 \\ y + 13z = -15 \\ y = -13z - 15 \\ \xrightarrow{X + (-13z - 15) - 3z = 4} \\ X = 16z + 19 \end{array} \\ \boxed{(16z + 19, -13z - 15, z)} \quad \boxed{\text{solution: } (16z + 19, -13z - 15, z)} \end{array}$$

$$\begin{cases} ax + by + cz = d \\ ax + by + cz = d \\ ax + by + cz = d \end{cases}$$

36. $m+n+t=9$
 $m-n-t=-15$
 $3m+n+t=2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -1 & -1 & -15 \\ 3 & 1 & 1 & 2 \end{array} \right] \xrightarrow{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -1 & -1 & -15 \\ 1 & 1 & 1 & 2 \end{array} \right]$$

$-1 + (-1)(1) = -2$
 $-1 + (-1)(1) = -2$
 $-15 - (-1)(9) = -24$

$1 + (-3)(1) = -2$
 $1 + (-3)(1) = -2$
 $2 + (-3)(9) = -25$

$R_2 + (-1)R_1$
 $R_3 + (-3)R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & -2 & -2 & -25 \end{array} \right] \xrightarrow{R_3 + (-2)R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & 0 & 0 & -1 \end{array} \right]$$

$0x + 0y + 0z = -1$
 $0 = -1$
 Contradiction!

NO Solution!

$R_1 + (-1)R_2$
 $R_3 + (-2)R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{\text{solution: } (1, 0, 1)}$$

$$\begin{array}{l} R_1 + (-1)R_3 \\ R_2 + (-1)R_3 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] \quad \text{solution: } (1, 0, 1)$$

10.1 Sequences and Series

An **infinite sequence** is a function that has as its domain the set of natural numbers.

$$\begin{array}{ccccccccc}
 1 & , & 2 & , & 3 & , & 4 & , & 5 & , & \dots & , & n & , & \dots \\
 \downarrow & & & & \downarrow \\
 2 & , & 4 & , & 8 & , & 16 & , & 32 & , & \dots & , & \dots \\
 2^1, 2^2, 2^3, 2^4, 2^5, \dots, 2^n
 \end{array}$$

What is the n^{th} term?

A **finite sequence** has as its domain $\{1, 2, 3, \dots, n\}$ for some n .

A **series** is a sum of terms of a sequence.

$$a_1 + a_2 + a_3 + \dots + a_n = \sum_{i=1}^n a_i$$

This notation is called "**sigma notation**" and is read as "the sum from i equals 1 to n of a sub i ."

$$2^1 + 2^2 + 2^3 + 2^4 + 2^5 = \sum_{i=1}^5 2^i$$

$$n^{\text{th}} \text{ term} = 2^n$$

$$2+4+8+\dots = \sum_{i=1}^{\infty} 2^i$$

$$\sum_{n=1}^{\infty} 2^n$$

$$\sum_{i=7}^{234} 2^i + \sum_{i=50,000}^{50,025} 2^i$$

10.1

$$2. a_n = (n-1)(n-2)(n-3)$$

$$a_1 = (1-1)(1-2)(1-3) = 0$$

$$a_2 = (2-1)(2-2)(2-3) = 0$$

$$a_3 = (3-1)(3-2)(3-3) = 0$$

$$a_4 = (4-1)(4-2)(4-3) = 3 \cdot 2 \cdot 1 = 6$$

$$a_5 = (5-1)(5-2)(5-3) = 4 \cdot 3 \cdot 2 = 24$$

$$0, 0, 0, 6, 24, \dots$$

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, ...
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 1 2 3 4 5 6 7 8 9 , ...

Recursively-defined sequence
 defines an element of the sequence
 in terms of other elements.

$$F_n = F_{n-1} + F_{n-2}$$

HW #9 (due Mon, Oct 20)

solve systems of equations in 2 and 3 variables using a matrix to get a single, integer solution:

- 8.1 #17, 19, 21 (but solve using a matrix)
- 8.3 #27, 31
- 8.2 #3*, 7, 9 (but solve using a matrix)

*some books have a **typo** on #3; the third equation *should* read:
 $2x+y+z=-3$

HW #10 (due Fri, Oct 24)

- 10.1 #7, 9, 23-31 odd
 #59, 63, 67
- 10.2 #9, 15, 19, 21, 25, 29
 #35, 37
- 10.3 #15, 19, 21, 25, 35, 37, 43, 45, 49, 57
- 10.7 #1, 7, 21, 27, 31-39 odd
- Final Exam Practice Problems