Topics / Problem Types that will appear on the Final Exam

From Test #1:
- Construct and simplify the difference quotient.
- Graph a basic algebraic function using transformations.
- Given a quadratic, determine the vertex of the parabola, axis of symmetry, increasing/decreasing, max/min, etc.

From Test #2:
- Graph a polynomial (and find all the zeros, multiplicities, end behavior, y-intercept, etc.)
- Graph a rational function (and find zeros, asymptotes, etc.)
- Solve a polynomial or rational inequality.

From Test #3:
- Given a one-to-one function, determine its inverse.
- Rewrite logarithmic expressions using product, quotient, power, and change of base rules.
- Solve logarithmic and exponential equations.

Since Test #3:
- Given a set with n elements, determine the number of ways to choose k of them.
- Find the nth term of an arithmetic and/or geometric sequence.
- Find the sum of the first n terms of an arithmetic or geometric series.
- Find the sum of an infinite geometric series.
- Find the (k+1)st term of a binomial.
- Solve a system of linear equations in three variables using matrices.

Bring your textbooks to the Exam.

$$\boldsymbol{a_{n+1} = a_n + d}$$

$$\boldsymbol{a_n = a_1 + (n-1)d}$$

$$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$$

$$\boldsymbol{a_n = a_1 r^{n-1}}$$

$$\boldsymbol{S_n = \frac{a_1(1-r^n)}{1-r}}$$

$$\boldsymbol{S_\infty = \frac{a_1}{1-r}}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$P(t) = P_0 e^{kt}$$

HW #11 (due Mon, Oct 27)
- 10.7 #1,7,21,27,31-39odd (Binomial Theorem)
- Final Exam Practice Problems

Review session - 3:30pm, Thursday 10/30

Final Exam - 9:00am, Friday 10/31
Note that your final exam average can replace your lowest test grade!

BRING YOUR TEXTBOOKS TO THE FINAL EXAM!

Solve:

$a+b-c=7$
$a-b+c=5$
$3a+b-c=-1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 1 & 5 \\ 3 & 1 & -1 & -1 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & -2 & 2 & -22 \end{array}\right]$$

$R_3 - R_2$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & 0 & -29 \end{array}\right]$$

$0 = -29$ ↯ no solution

1. Find the indicated term of the sequence.
   $a_n = (n-43)^{n+1}$; a_5

$a_5 = (5-43)^{5+1} = (-38)^6 = \boxed{3010936384}$

2. Predict the nth term of the sequence.
   $\frac{2}{2^3}, \frac{3}{2^4}, \frac{4}{2^5}, \frac{5}{2^6}, \frac{6}{2^7}, \ldots$

$a_n = \frac{n+1}{2^{n+2}}$

3. Find and evaluate the sum.

$$\sum_{k=4}^{25} 3k = 3\cdot 4 + 3\cdot 5 + 3\cdot 6 + 3\cdot 7 + \ldots + 3\cdot 25$$

$$= 12 + 15 + 18 + 21 + \ldots + 75$$

arithmetic series w/ common difference of 3

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{22}{2}(12 + 75) = 11(87) = \boxed{957}$$

4. Write sigma notation for the series.

$$4 + 9 + 16 + 25 + \cdots + 144$$

$$2^2 + 3^2 + 4^2 + 5^2 + \cdots + 12^2$$

$$\boxed{\sum_{i=2}^{12} i^2}$$

$$\sum_{i=1}^{11} (i+1)^2$$

Write solutions to the systems of equations described by the following matrices:

5. $\left[\begin{array}{ccc|c} 1 & 0 & 5 & -9 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 3 \end{array}\right]$

no solution

6. $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 7 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$1x + 0y - 3z = 7 \Rightarrow x = 3z + 7$

$0x + 1y + 2z = -5 \Rightarrow y = -2z - 5$

$(x, f(x), g(x))$

$(f(y), y, g(y))$

$(f(z), g(z), z)$

$$\boxed{(3z+7, -2z-5, z)}$$

7. Solve the system of equations. *Solution should be integers.*

$$\begin{cases} 2x + y + z = -3 \\ x - 2y + 3z = 6 \\ x + y + z = 6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & -3 \\ 1 & -2 & 3 & 6 \\ 1 & 1 & 1 & 6 \end{array}\right] \xrightarrow[\text{R1 and R3}]{\text{swap rows}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -2 & 3 & 6 \\ 2 & 1 & 1 & -3 \end{array}\right]$$

$$\xrightarrow[R3+(-2)\cdot R1]{R2+(-1)\cdot R1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 2 & 0 \\ 0 & -1 & -1 & -15 \end{array}\right] \xrightarrow{R2\cdot(-1/3)} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2/3 & 0 \\ 0 & -1 & -1 & -15 \end{array}\right]$$

$$\xrightarrow[R3+(1)\cdot R2]{R1+(-1)\cdot R2} \left[\begin{array}{ccc|c} 1 & 0 & 5/3 & 6 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & -5/3 & -15 \end{array}\right] \xrightarrow{R3\cdot(-3/5)} \left[\begin{array}{ccc|c} 1 & 0 & 5/3 & 6 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\xrightarrow[R2+(2/3)\cdot R3]{R1+(-5/3)\cdot R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$\boxed{\textit{solution: } (-9, 6, 9)}$

Find the 4th term of $(2x - y)^{12}$.

$(k+1)^{st}$

$n = 12$

$k = 3$

$a = 2x$

$b = y$

$$\binom{12}{3}(2x)^{12-3}(-y)^3$$

$$\frac{12!}{3!(12-3)!}(2x)^9(-y)^3$$

$$\frac{\cancel{12}^{2}\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{3\cdot 2}\cdot 1\cdot \cancel{9!}}\, 2^9 x^9 (-y)^3$$

$$-2^{10}(110)x^9y^3 = \boxed{-112640x^9y^3}$$

$$\binom{n}{k}a^{n-k}b^k$$

$\uparrow$

$(k+1)^{st}$ term of $(a+b)^n$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$(2-3i)^7$ 3^{rd} term

$a=2$

$b=-3i$

$n=7$

$k=2$

$\binom{7}{2}(2)^{7-2}(-3i)^2$

$\frac{7!}{2!(7-2)!}(2^5)(9)(i^2)$ $=-1$

$\frac{7\cdot \cancel{6}^{3}\cdot \cancel{5!}}{\cancel{2}\cdot 1\cdot \cancel{5!}}(-32\cdot 9)$

$(21)(-32)(9)=$

$i^{57}=i^{56}\cdot i$

$=(i^4)^{14}i$

$=i$

For the functions $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x-3}$,

1. find $(f \circ g)(x)$

$$= \frac{1}{\sqrt{x-3}}$$

$$x - 3 > 0$$

2. give the domain of $(f \circ g)(x)$

$$\{x \mid x > 3\} = (3, \infty)$$

3. find $(g \circ f)(x)$

$$= \sqrt{\frac{1}{x} - 3}$$

$$\frac{1 - 3(-2)}{-2} \quad +6$$

$$\frac{1}{x} - 3 \geq 0 \qquad -3\left(x - \tfrac{1}{3}\right)$$

$$\frac{1 - 3x}{x} \geq 0$$

4. give the domain of $(g \circ f)(x)$

$$\left(0, \tfrac{1}{3}\right]$$

$-$ 0 $+$ $\frac{1}{3}$ $-$

Write a slope-intercept equation $(y = mx + b)$ for a line passing through the given point that is perpendicular to the given line.

$$(3, 5), \qquad y = \frac{2}{7}x + 1$$

$$m = -\frac{7}{2}$$

$$(y - 5) = -\frac{7}{2}(x - 3)$$

$$y = -\frac{7}{2}x + \frac{21}{2} + \frac{10}{2}$$

$$\boxed{y = -\frac{7}{2}x + \frac{31}{2}}$$

Solve for x.

1. $5^{4x-7} = 125$

2. $\log x + \log(x+4) = \log 12$

Find the sum of the geometric series, if it exists. Give an exact answer.

$$-8 + 4 + (-2) + \cdots$$

Given a set with 9 elements, how many ways are there to choose 5 of them?

For the graph of the function $f(x) = -4x^2 + 24x - 20$

a. Find the vertex.

b. State the equation of the axis of symmetry.

c. State the interval(s) on which the function is increasing.

d. State the interval(s) on which the function is decreasing.

e. State the y-intercept as an ordered pair.

f. State the x-intercept(s) (if any) as ordered pairs.

Construct and simplify the difference quotient for $f(x) = 5x^2 + 3x$.

1. Find the 32^{nd} term of the arithmetic sequence $92, 87, 82, 77, 72, \ldots$

2. Determine the sum of the first 19 terms of the geometric series $-81 + 27 - 9 + 3 - 1 + \cdots$

3. Find and evaluate the sum.
$$\sum_{k=1}^{\infty} \left(\frac{3}{5}\right)^k$$

4. Write sigma notation for the series. $4 - 9 + 16 - 25 + \cdots + (-1)^n n^2$

5. Write the 4^{th} term of $(2x - y)^6$.

Find a formula for the inverse of the one-to-one function.

$$f(x) = \frac{x - 1}{x + 7}$$

Graph the following piecewise function by hand, and state on which intervals f is increasing, decreasing, and constant.

$$f(x) = \begin{cases} 4, & for\ x \leq -2 \\ x+1, & for - 2 < x < 3 \\ -x, & for\ x \geq 3 \end{cases}$$

3. $x - y + 2z = -3$
$x + 2y + 3z = 4$
$2x + y + z = -3$

ans: $(-3, 2, 1)$

7. $x + 2y - z = -8$
$2x - y + z = 4$
$8x + y + z = 2$

ans: no solution

9. $2x + y - 3z = 1$
$x - 4y + z = 6$
$4x - 7y - z = 13$

ans: $\left(\frac{11y+19}{5}, y, \frac{9y+11}{5}\right)$