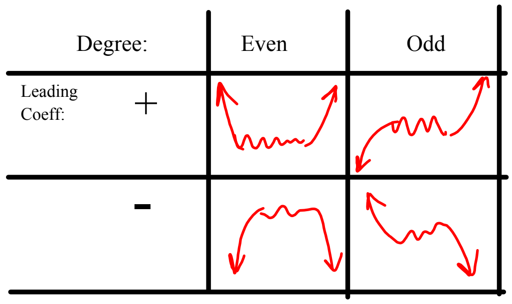
3.1:	#8-14 all	Describing simple characteristics of polynomials
	#23-31 all	Determining zeros & multiplicities from factored polynomials
3.2:	#16,17,21,22,24,25,27,28	Graph polynomials that are already factored
3.1:	#32-38	Finding zeros & multiplicities of expanded polynomials
3.2:	#18-20,23,26,29-32	Graphing expanded polynomials
3.3:	#7,9,13,19,21,23, 35	Long & synthetic polynomial division
3.4:	#7-16all	Given the zeros of a polynomial, find the polynomial
	#25-32all; 43-47odd	Given some zeros of a polynomial, find the other zeros
	#51-54all	List all possible rational zeros
	#55-69odd	Find all the zeros and write f(x) in factored form
	#79,89,93	Descartes' rule of signs
	#95-98all	Graph the polynomial

3.1/3.2 - Polynomial Functions

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

$a_n x^n$ is the lead term
a_n is the leading coefficient
n is the degree of the polynomial
a_0 is the constant term

If leading coefficient is negative, vertical flip!



The graph of an n^{th} degree polynomial has at most n-1 turning points.

Examples:

cubic: 6^{th} degree:

zeros of a function v. x-intercepts of a function

x is a zero of a function *f* if *f(x)=0*
That is, zeros of a function are all of the input values that have 0 as their output.

(x,0) is an x-intercept of a function *f* if the graph of *f* intersects the x-axis at the point *(x,0)* .

Since the x-coordinates of the x-intercepts are exactly those input values that map to 0, we can get our x-intercepts from our list of zeros.
Note, however, that *only real zeros contribute to x-intercepts*

The degree of a polynomial determines the number of zeros it has:

The Fundamental Theorem of Algebra

An n^{th} degree polynomial has n zeros (not necessarily unique), and can be written as the product of n linear factors.

$$f(x) = (x-b_1)(x-b_2)\cdots(x-b_n)$$

Multiplicity of a zero

$$f(x) = (x-2)^4(x-3)(x+6)^3(x+2)^2$$

$$y=(x-2)^4(x+3)^3$$

$$f(x)=(x-2)^4(x-3)(x+6)^3(x+2)^2$$

$$f(x) = -3x^2(x-1)^3(x+2)^2$$

3.1/3.2 - Polynomial Functions

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$$

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n is the degree of the polynomial
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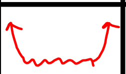
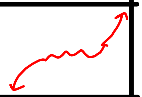
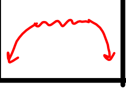
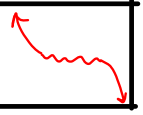
Lead Term Test

	Degree: Even	Odd
Leading Coefficient +		
-		

The degree of a polynomial determines the number of zeros it has:

The Fundamental Theorem of Algebra
An n^{th} degree polynomial has n zeros (not necessarily unique), and can be written as the product of n linear factors.

$$f(x) = (x-b_1)(x-b_2)\cdots(x-b_n)$$

The graph of an n^{th} degree polynomial has at most n-1 turning points.

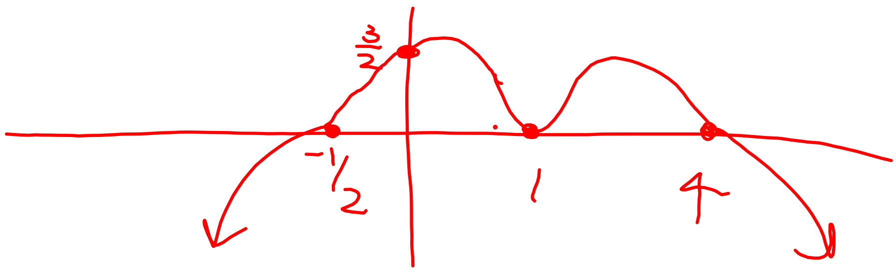
$$f(x) = -3(x-4)^1(x+\tfrac{1}{2})^3(x-1)^2$$

lead term: $-3 \cdot x^1 \cdot x^3 \cdot x^2 = -3x^6$

y-int: $f(0) = -3(-4)(\frac{1}{2})^3(-1)^2 = -3(-4)\cdot\frac{1}{8} = \frac{3}{2}$

zeros: 4, $-\frac{1}{2}$, 1

mult: odd, odd, even



$$f(x) = -2x^5 - x^3 = -x^3(2x^2+1)$$

(real) zeros: 0 (mult 3)

y-int: (0,0)

lead term: $-2x^5$

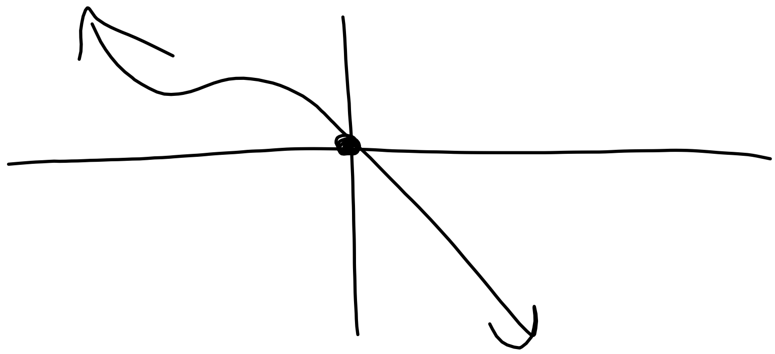
$2x^2+1=0$

$2x^2=-1$

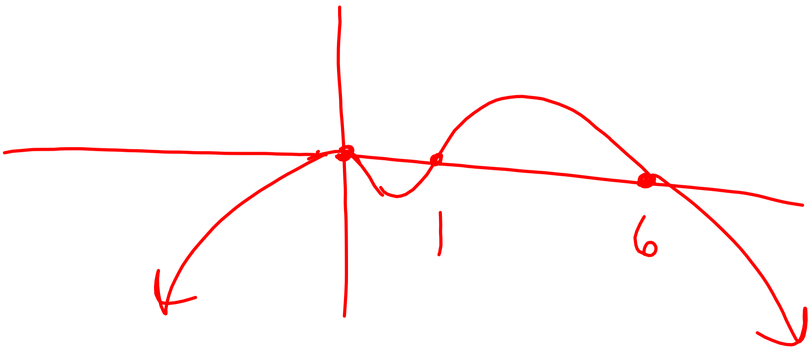
$x^2=-\frac{1}{2}$

$x=\pm\sqrt{\frac{-1}{2}}$

$=\pm\frac{i}{\sqrt{2}}$



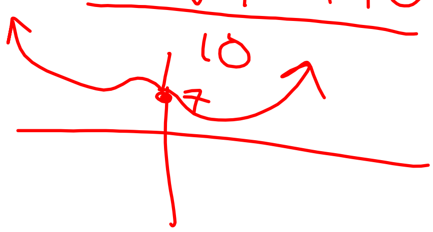
$$y=-x^4+7x^3-6x^2 = -x^2(x^2-7x+6)$$
$$= -x^2(x-6)(x-1)$$



$$f(x) = 5x^4-3x^2+7 \qquad \text{Let } u=x^2$$
$$= \underbrace{5u^2-3u+7}_{5(7)=35}$$

$$u = \frac{-(-3) \pm \sqrt{(-3)^2-4(5)(7)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9-140}}{10} = \frac{3 \pm i\sqrt{131}}{10} = x^2$$

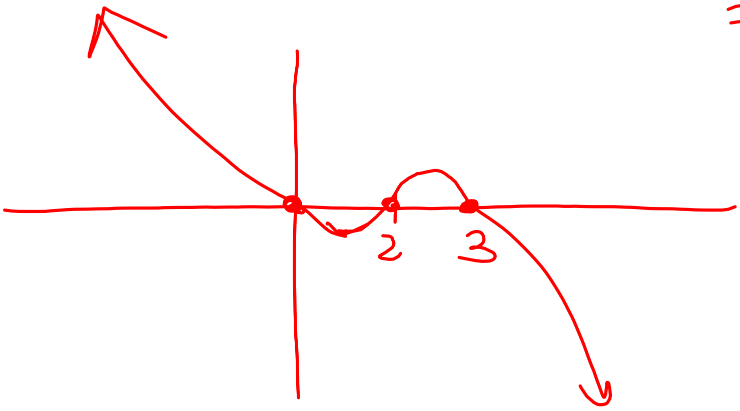


$$\pm\sqrt{\frac{3 \pm i\sqrt{131}}{10}} = x$$

4 complex zeros

$f(x) = -x^5 + 5x^4 - 6x^3 = -x^3(x^2 - 5x + 6)$

$= -x^3(x-2)(x-3)$



3.3 Zeros of Polynomials &
Polynomial Division

$f(x) = (x-a)(x-b)(x-c)$

$\Rightarrow$ zeros are $a, b, \& c$.

$12 \div 3 = 4 \Rightarrow 12 = 3 \cdot 4 = 3 \cdot 2 \cdot 2$

Long Division

3.3 #6

$P(x) = 2x^3 - 3x^2 + x - 1$

$d(x) = x - 3$

$\frac{P(x)}{d(x)} = P(x) \div d(x)$

$5 \div 2 =$

$$\begin{array}{r} 2 \\ 2\overline{)5} \\ -4 \\ \hline 1 \end{array} \quad = 2 \cdot 2 + 1$$

$$\begin{array}{r} 2x^2 + 3x + 10 \\ x-3\overline{)2x^3 - 3x^2 + x - 1} \\ -(2x^3 - 6x^2) \\ \hline 3x^2 + x - 1 \\ -(3x^2 - 9x) \\ \hline 10x - 1 \\ -(10x - 30) \\ \hline 29 \end{array}$$

$P(x) = (x-3)\underbrace{(2x^2+3x+10)}_{\text{quotient}} + \underbrace{29}_{\text{remainder}}$

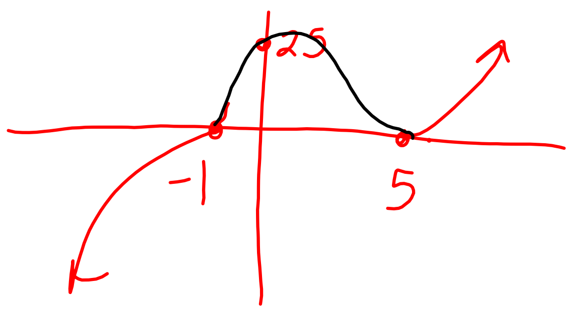
8. $P(x) = x^3 - 9x^2 + 15x + 25 = (x-5)(x^2-4x-5)$

$d(x) = x - 5$

$= (x-5)(x-5)(x+1)$

$= (x-5)^2(x+1)$

$$\begin{array}{r} x^2 - 4x - 5 \\ x-5\overline{)x^3 - 9x^2 + 15x + 25} \\ -(x^3 - 5x^2) \\ \hline -4x^2 + 15x + 25 \\ -(-4x^2 + 20x) \\ \hline -5x + 25 \\ -(-5x + 25) \\ \hline 0 \end{array}$$



Synthetic Division

12. $(x^3 - 7x^2 + 13x + 3) \div (x-2)$

Quotient: $x^2 - 5x + 3$

Remainder: 9

$$
\begin{array}{r|rrrr}
2 & 1 & -7 & 13 & 3 \\
 & \downarrow & 2 & -10 & 6 \\
\hline
 & 1 & -5 & 3 & \boxed{9} \\
 & \uparrow & \uparrow & \uparrow & \uparrow \\
 & x^2 & x & \text{const.} & \text{remainder}
\end{array}
$$

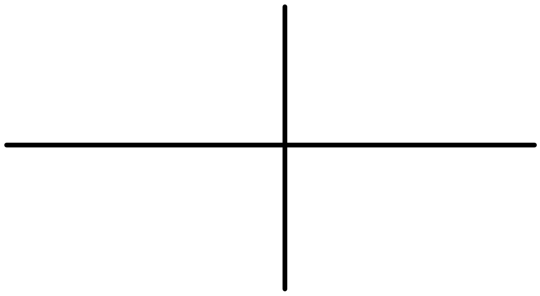
18. $(x^7 - x^6 + x^5 - x^4 + 2) \div (x+1)$

$$
\begin{array}{r|rrrrrrrr}
 & & & & & \text{missing } x^3 & x^2 & x & \\
-1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 2 \\
 & \downarrow & -1 & 2 & -3 & 4 & -4 & 4 & -4 \\
\hline
 & 1 & -2 & 3 & -4 & 4 & -4 & 4 & \boxed{-2}
\end{array}
$$

Quotient: $x^6 - 2x^5 + 3x^4 - 4x^3 + 4x^2 - 4x + 4$

Remainder: -2

Graph: $f(x) = -6\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{3}\right)^2(x+2)$



zeros:

multiplicity:

y-intercept:

lead term:

20. $(x^5+32) \div (x+2)$

32. $f(x)=3x^3+11x^2-2x+8$

are the #'s -4 & 2 zeros of the function?

$-4|$ 3 11 -2 8

$2|$ 3 11 -2 8

46. $f(x)=x^4-4x^3-7x^2+34x-24$

Factor the polynomial and solve $f(x)=0$.

<u>3.3</u>

38. Determine if i or $-i$ are zeros of the polynomial $f(x) = x^3 + 2x^2 + x + 2$

If the discriminant $b^2 - 4ac < 0$,
we know that the quadratic function has two complex conjugate zeros.

This means that if i is a zero, then so is $-i$, and in general,
if $\boldsymbol{a + bi}$ is a zero, then so is $\boldsymbol{a - bi}$.

If $f(x)$ has $a + bi$ and $a - bi$ as its only zeros, then

$f(x) = [x - (a + bi)][x - (a - bi)]$

$f(x) = x^3 + 2x^2 + x + 2$

Knowing that i and $-i$ are zeros of f, determine the third and final zero.

$a + \sqrt{b}$ and $a - \sqrt{b}$ also come in conjugate pairs.

If 2, $\sqrt{3}$, and $2 - i$ are zeros of the polynomial $p(x)$, then

$p(x) =$

Rational Zeros Theorem

Given a polynomial $p(x) = a_n x^n + \cdots + a_1 x + a_0$,

The only possible rational zeros are of the form

$$\pm \frac{factors\ of\ a_0}{factors\ of\ a_n}$$

$p(x) = 4x^3 - 7x^2 + 21x - 5$

Possible rational zeros:

3.4

74. Find all the zeros of the polynomial $f(x) = 2x^3 + 3x^2 + 2x + 3$

Descartes' Rule of Signs

If $P(x)$ is written in descending order with real numbered coefficients and a nonzero constant term*, then

- The number of positive real zeros is either
  - the number of sign changes of $P(x)$ or
  - less than that number by a positive even integer.
- The number of negative real zeros is either
  - the number of sign changes of $P(-x)$ or
  - less than that number by a positive even integer.

* if the constant term is zero, factor out x (or GCF), and use Descartes' rule of signs on whatever's leftover

3.4

84. $H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$

80. $g(x) = 5x^6 - 3x^3 + x^2 - x$

86. $g(z) = -z^{10} + 8z^7 + z^3 + 6z - 1$

Factor:

$f(x) = x^3 + 3x^2 - 2x - 6$

3.4

Find all the zeros and write f(x) in factored form.

63. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

$\underline{-1|}$ 1 -3 -20 -24 -8

$\underline{-2|}$

3.4

Graph the polynomial.

96. $f(x) = 3x^3 - 4x^2 - 5x + 2$

$\underline{-1|}$ 3 -4 -5 2

3.4

Graph the polynomial.

98. $f(x) = 3x^4 - 37x^2 + 9$