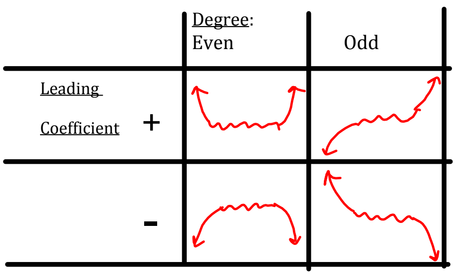
| | |
|---|---|
| 3.1: #8-14 all | Describing simple characteristics of polynomials |
| #23-31 all | Determining zeros & multiplicities from factored polynomials |
| 3.2: #16,17,21,22,24,25,27,28 | Graph polynomials that are already factored |
| 3.1: #32-38 | Finding zeros & multiplicities of expanded polynomials |
| 3.2: #18-20,23,26,29-32 | Graphing expanded polynomials |
| 3.3: #7,9,13,19,21,23, 35 | Long & synthetic polynomial division |
| 3.4: #7-16all | Given the zeros of a polynomial, find the polynomial |
| #25-32all; 43-47odd | Given some zeros of a polynomial, find the other zeros |
| #51-54all | List all possible rational zeros |
| #55-69odd | Find all the zeros and write f(x) in factored form |
| #79,89,93 | Descartes' rule of signs |
| #95-98all | Graph the polynomial |

# 3.1/3.2 - Polynomial Functions

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

**$a_n x^n$** is the lead term
**$a_n$** is the leading coefficient
**n** is the degree of the polynomial
**$a_0$** is the constant term

## Lead Term Test




The degree of a polynomial determines the number of zeros it has:

**The Fundamental Theorem of Algebra**
An n^th degree polynomial has n zeros (not necessarily unique), and can be written as the product of n linear factors.

$$f(x) = (x-b_1)(x-b_2)\cdots(x-b_n)$$

The graph of an n^th degree polynomial has at most n-1 turning points.

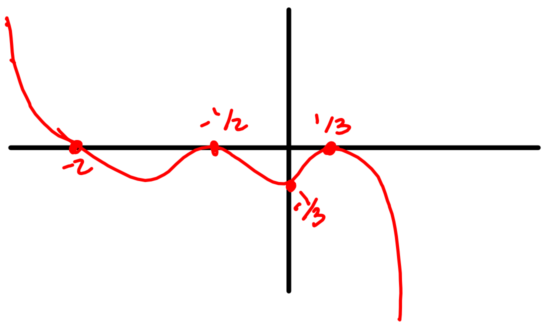
## zeros of a function v. x-intercepts of a function

$x$ is a zero of a function $f$ if $f(x)=0$
That is, zeros of a function are all of the input values that have 0 as their output.

$(x,0)$ is an x-intercept of a function $f$ if the graph of $f$ intersects the x-axis at the point $(x,0)$ .

Since the x-coordinates of the x-intercepts are exactly those input values that map to 0, we can get our x-intercepts from our list of zeros.
Note, however, that *only real zeros contribute to x-intercepts*

Graph: $f(x) = -6\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{3}\right)^2(x+2)$




zeros: $-\frac{1}{2}$ ; $\frac{1}{3}$ ; $-2$

multiplicity: 2 ; 2 ; 1

y-intercept: $-\frac{1}{3}$ $\quad -\frac{6}{1}\cdot\frac{1}{4}\cdot\frac{1}{9}\cdot\frac{2}{1} = -\frac{12}{36} = -\frac{1}{3}$

lead term: $-6x^5$

20. $(x^5+32) \div (x+2)$

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & 0 & 0 & 32 \\ & & -2 & 4 & -8 & 16 & -32 \\ \hline & 1 & -2 & 4 & -8 & 16 & \boxed{0} \end{array}$$

$x^5+32=(x+2)(x^4-2x^3+4x^2-8x+16)$

32. $f(x)=3x^3+11x^2-2x+8$

are the #'s $-4$ & $2$ zeros of the function?

$$\begin{array}{r|rrrr} -4 & 3 & 11 & -2 & 8 \\ & & -12 & 4 & -8 \\ \hline & 3 & -1 & 2 & \boxed{0} \end{array}$$

yes

$f(x)=(x+4)(3x^2-x+2)$

$$\begin{array}{r|rrrr} 2 & 3 & 11 & -2 & 8 \\ & & 6 & 34 & 64 \\ \hline & 3 & 17 & 32 & \boxed{72} \end{array}$$

no

46. $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

Factor the polynomial and solve $f(x) = 0$.

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & -7 & 34 & -24 \\ & & 4 & 0 & -28 & 24 \\ \hline & 1 & 0 & -7 & 6 & \boxed{0} \end{array}$$

$f(x) = (x-4)(x^3 - 7x + 6)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

$f(x) = (x-4)(x-1)(x^2 + x - 6)$

$\boxed{f(x) = (x-4)(x-1)(x-2)(x+3)}$

zeros: 4, 1, 2, -3

3.3 $\quad i = \sqrt{-1} \quad i^2 = -1$

38. Determine if $i$ or $-i$ are zeros of the polynomial $f(x) = x^3 + 2x^2 + x + 2$

$$\begin{array}{r|rrrr} i & 1 & 2 & 1 & 2 \\ & & & i(2+i) & i(2i) \\ & & i & 2i-1 & -2 \\ \hline & 1 & 2+i & 2i & \boxed{0} \end{array}$$ yes

$$\begin{array}{r|rrrr} -i & 1 & 2 & 1 & 2 \\ & & & -i(2-i) & -i(-2i) \\ & & -i & -2i-1 & -2 \\ \hline & 1 & 2-i & -2i & \boxed{0} \end{array}$$ yes

If the discriminant $b^2 - 4ac < 0$,
we know that the quadratic function has two complex conjugate zeros.

This means that if $i$ is a zero, then so is $-i$, and in general,
**if $a + bi$ is a zero, then so is $a - bi$.**

If $f(x)$ has $a + bi$ and $a - bi$ as its only zeros, then

$$f(x) = [x - (a + bi)][x - (a - bi)]$$

$$[x - a - bi][x - a + bi]$$

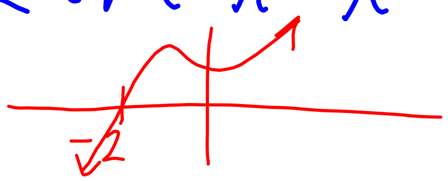
$f(x) = x^3 + 2x^2 + x + 2$
Knowing that $i$ and $-i$ are zeros of $f$, determine the third and final zero.

$$(x-i)(x+i) = x^2 + ix - ix - i^2 = x^2 - (-1) = x^2 + 1$$

$$\begin{array}{r|l} & x + 2 \\ \hline x^2+1 & x^3 + 2x^2 + x + 2 \\ & -(x^3 + \quad x) \\ \hline & 2x^2 + 2 \\ & -(2x^2 + 2) \\ \hline & 0 \end{array}$$

zeros: $\pm i, -2$

$f(x) = (x-i)(x+i)(x+2)$



**$a + \sqrt{b}$ and $a - \sqrt{b}$ also come in conjugate pairs.**

$-\sqrt{3}$ $2+i$

If 2, $\sqrt{3}$, and $2 - i$ are zeros of the polynomial $p(x)$, then

$$p(x) = (x-2)(x-\sqrt{3})(x-(-\sqrt{3}))(x-(2-i))(x-(2+i))$$

**Rational Zeros Theorem**

leading coeff. constant term

Given a polynomial $\underbrace{p(x) = \boxed{a_n} x^n + \cdots + a_1 x + \boxed{a_0}}$,

The only possible rational zeros are of the form

$$\pm \frac{factors\ of\ a_0}{factors\ of\ a_n}$$

$a_n$ $a_0$

$p(x) = 4x^3 - 7x^2 + 21x - 5$

Possible rational zeros:

$$\pm \frac{1, 5}{1, 2, 4} = 1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}, 5, -5, \frac{5}{2}, -\frac{5}{2}, \frac{5}{4}, -\frac{5}{4}$$

3.4

74. Find all the zeros of the polynomial $f(x) = 2x^3 + 3x^2 + 2x + 3$

possible rational zeros:

$\pm \frac{1, 3}{1, 2} = \pm 1, \pm \frac{1}{2}, \cancel{\pm 3}, \pm \frac{3}{2}$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & 2 & 3 \\ & & -6 & 9 & -33 \\ \hline & 2 & -3 & 11 & -30 \end{array}$$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 3 & 2 & 3 \\ & \downarrow & -3 & 0 & -3 \\ \hline & 2 & 0 & 2 & \boxed{0} \end{array}$$

$f(x) = (x + \frac{3}{2})(2x^2 + 2)$

$= 2(x + \frac{3}{2})(x^2 + 1)$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

$f(x) = 2(x + 3/2)(x + i)(x - i)$

zeros: $-3/2, \pm i$

**Descartes' Rule of Signs**

If $P(x)$ is written in descending order with real numbered coefficients and a nonzero constant term,* then

- The number of positive real zeros is either
  - the number of sign changes of $P(x)$ or
  - less than that number by a positive even integer.
- The number of negative real zeros is either
  - the number of sign changes of $P(-x)$ or
  - less than that number by a positive even integer.

* if the constant term is zero, factor out x (or GCF), and use Descartes' rule of signs on whatever's leftover

## 3.4

84. $H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$

$+ \quad - \quad + \quad + \quad +$

2 sign changes $\Rightarrow$ either 2 or 0 positive real zeros

$H(-t) = 5(-t)^{12} - 7(-t)^4 + 3(-t)^2 + (-t) + 1$

$= 5t^{12} - 7t^4 + 3t^2 - t + 1$

$+ \quad - \quad + \quad - \quad +$

4 sign changes $\Rightarrow$ either 4 or 2 or 0 negative real zeros

80. $g(x) = 5x^6 - 3x^3 + x^2 - x + 0$

$= x(\underbrace{5x^5 - 3x^2 + x - 1}_{f(x)})$

$x = 0$ is a zero

$f(x)$ has 3 sign changes $\Rightarrow$ 3 or 1 positive real zeros

$+ \quad - \quad + \quad -$

$f(-x) = -5x^5 - 3x^2 - x - 1$

0 sign changes $\Rightarrow$ 0 negative real zeros