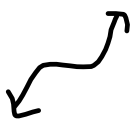
Precalculus - Quiz 2 Solutions & 3.4-3.5 Review — December 12, 2016

3.1: #8-14 all	Describing simple characteristics of polynomials
#23-31 all	Determining zeros & multiplicities from factored polynomials
3.2: #16,17,21,22,24,25,27,28	Graph polynomials that are already factored
3.1: #32-38	Finding zeros & multiplicities of expanded polynomials
3.2: #18-20,23,26,29-32	Graphing expanded polynomials
3.3: #7,9,13,19,21,23, 35	Long & synthetic polynomial division
3.4: #7-16all	Given the zeros of a polynomial, find the polynomial
#25-32all; 43-47odd	Given some zeros of a polynomial, find the other zeros
#51-54all	List all possible rational zeros
#55-69odd	Find all the zeros and write f(x) in factored form
#79,89,93	Descartes' rule of signs
#95-98all	Graph the polynomial
3.5: #7-25odd	**Determining asymptotes of rational functions**
#27-67odd	**Graphing rational functions**
3.6 #15-39odd	**Solving polynomial inequalities**
#47, 53-61odd	**Solving rational inequalities**
3.7: #23-37 odd	**Variation**

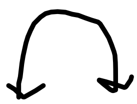
Sketch the end behavior for the polynomial with the given lead term.

1. $5x^4$

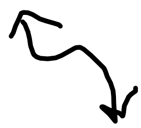
2. $5x^7$



3. $-2x^6$



4. $-4x^5$



For the polynomial $f(x) = -2(x-1)^3(x-2)^2(x+3)$,

5. Determine the lead term.

$-2x^3x^2x = -2x^6$

6. Determine the y-intercept.

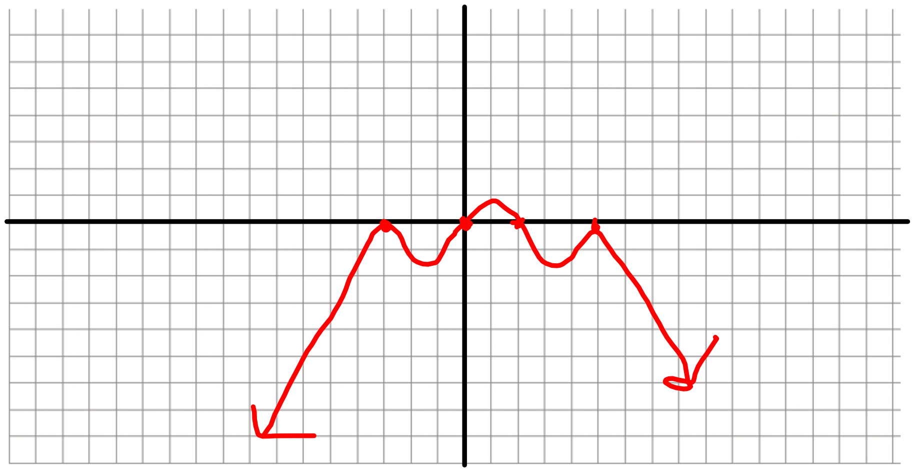
$-2(-1)^3(-2)^2(3)$ $(0,24)$

$= -2(-1)(4)(3) = 24$

7. Given the information about the polynomial function, draw its graph.

Lead term: $-5x^{10}$

Zeros:	0	2	5	-3
Multiplicity:	3	1	2	2



Which of the following are ***not*** possible rational zeros of the polynomial $(x) = 3x^5 - 4x^4 + 2x^3 - x - 4$ according to the Rational Zeros Theorem?
(circle your answer(s))

$$\pm \frac{1, 2, 4}{1, 3}$$

$a.\ \frac{2}{3}$ $b. -1$ $c.\ 4$ $d.\ 3$ $e.\ -2$ $f. -\frac{3}{2}$

Which of the following does Descartes' Rule of Signs tell us about the zeros of the polynomial $f(x) = 3x^5 - 4x^4 + 2x^3 - x - 4$? **(circle your answer(s))**

$+ \quad - \quad + \quad - \quad -$ 3 or 1 pos.

$f(-x) = -3x^5 - 4x^4 - 2x^3 + x - 4$

$- \quad - \quad - \quad + \quad -$

2 or 0 neg.

a. it has 4 or 2 or 0 positive real zeros

b. it has 3 or 1 positive real zeros

c. it has 3 or 1 negative real zeros

d. it has 2 or 0 negative real zeros

e. it has 1 negative real zero

f. it has no negative real zero

11. Construct the polynomial of *least* degree that has the following as some of its zeros: 1 ; -2 ; $\sqrt{3}$; $2-3i$
(Please leave your polynomial in factored form!)

$-\sqrt{3} \quad 2+3i$

$f(x) =$

$(x-1)(x-(-2))(x-\sqrt{3})(x-(-\sqrt{3}))\cdot(x-(2-3i))(x-(2+3i))$

$= (x-1)(x+2)(x-\sqrt{3})(x+\sqrt{3})(x-2+3i)(x-2-3i)$

12. Given that 3 is a zero of the polynomial $f(x) = x^3 - 3x^2 + 4x - 12$, find all other zeros and write the polynomial as a product of linear factors.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 4 & -12 \\ & & 3 & 0 & 12 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array}$$

$x^2+4=0$

$x^2=-4$

$\boxed{x=\pm 2i}$

$f(x) = (x-3)(x^2+4)$

$= \boxed{(x-3)(x-2i)(x+2i)}$

Factor.

3.3

46. $f(x)=x^4-4x^3-7x^2+34x-24$

hint: 1 is a zero.

$\pm 1,2,3,4,6,8,12,24$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -7 & 34 & -24 \\ & & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & \boxed{0} \end{array}$$

$f(x)=(x-1)(x^3-3x^2-10x+24)$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$f(x)=(x-1)(x-2)(x^2-x-12)$

$\boxed{f(x)=(x-1)(x-2)(x-4)(x+3)}$

48. $f(x)=x^4+11x^3+41x^2+61x+30$

no positive real zeros

$f(-x)=x^4-11x^3+41x^2-61x+30$ 4 or 2 or 0 neg real zeros

$+ \quad - \quad + \quad - \quad +$

possible rational zeros: $-(1, \cancel{2}, 3, 5, \cancel{6}, \cancel{10}, 15, \cancel{30})$

-2	1	11	41	61	30
		-2	-18	-46	-30
	1	9	23	15	0

$f(x)=(x+2)(x^3+9x^2+23x+15)=(x+2)(x+3)(x^2+6x+5)$

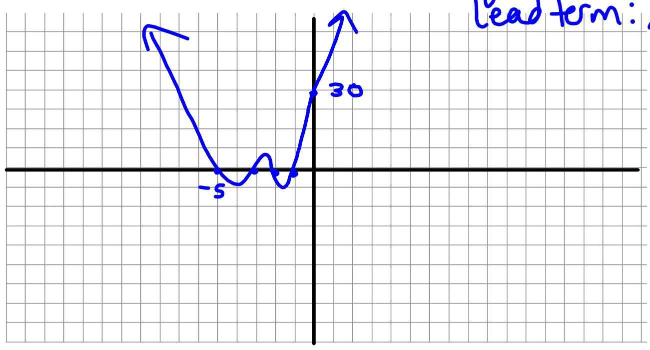
$=(x+2)(x+3)(x+5)(x+1)$

-3	1	9	23	15
		-3	-18	-15
	1	6	5	0

zeros: $-1, -2, -3, -5$

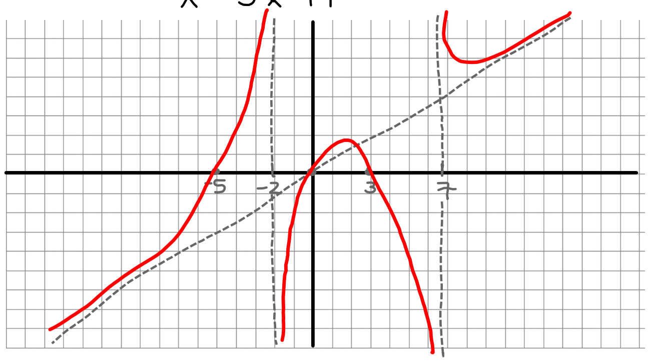
y-int: $(0, 30)$

Lead term: x^4



3.5

63. $f(x)=\dfrac{x^3+2x^2-15x}{x^2-5x-14}=\dfrac{x(x^2+2x-15)}{(x-7)(x+2)}=\dfrac{x(x+5)(x-3)}{(x-7)(x+2)}$



zeros: $0, -5, 3$

y-int: $(0,0)$

V.A: $x=7, x=-2$

end behavior: $\dfrac{x^3}{x^2}=x$

$y=x$

$$f(x)=\frac{\cancel{(x-2)}(x+4)(x-3)}{\cancel{(x-2)}(x+1)(x+2)}$$

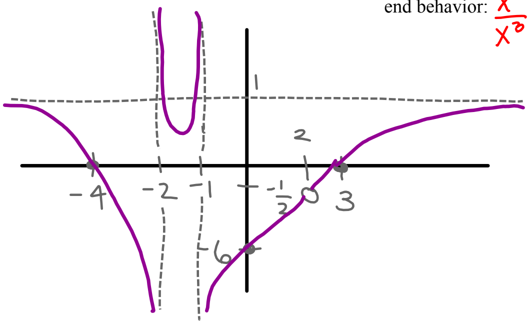
hole @ $\left(2, -\frac{1}{2}\right)$

zero(s): $-4, 3$

y-intercept: $(0, -6)$

vertical asymptote(s): $x=-1, x=-2$

end behavior: $\frac{x^3}{x^3}=1 \quad y=1$



Limits:

as x -->	f(x) -->
$-\infty$	1
-4	0
-1^+	$-\infty$
0	-6
2	$-1/2$
∞	1

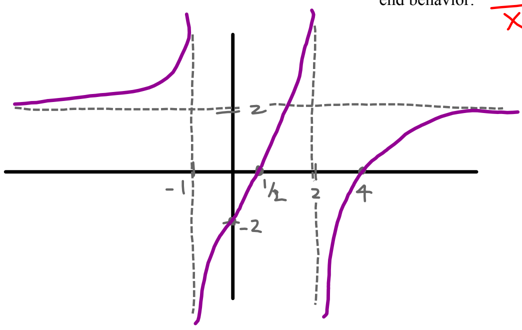
$$y=\frac{(2x-1)(x-4)}{(x-2)(x+1)}$$

zero(s): $1/2, 4$

y-intercept: $(0, -2)$

vertical asymptote(s): $x=2, x=-1$

end behavior: $\frac{2x^2}{x^2}=2 \quad y=2$



Limits:

as x -->	f(x) -->
$-\infty$	2
-1^-	∞
0	-2
$1/2$	0
2^+	$-\infty$
∞	2