3.1: #8-14 all	Describing simple characteristics of polynomials
#23-31 all	Determining zeros & multiplicities from factored polynomials
3.2: #16,17,21,22,24,25,27,28	Graph polynomials that are already factored
3.1: #32-38	Finding zeros & multiplicities of expanded polynomials
3.2: #18-20,23,26,29-32	Graphing expanded polynomials
3.3: #7,9,13,19,21,23, 35	Long & synthetic polynomial division
3.4: #7-16all	Given the zeros of a polynomial, find the polynomial
#25-32all; 43-47odd	Given some zeros of a polynomial, find the other zeros
#51-54all	List all possible rational zeros
#55-69odd	Find all the zeros and write f(x) in factored form
#79,89,93	Descartes' rule of signs
#95-98all	Graph the polynomial

3.5: #7-25odd	**Determining asymptotes of rational functions**
#27-67odd	**Graphing rational functions**
3.6 #15-39odd	**Solving polynomial inequalities**
#47, 53-61odd	**Solving rational inequalities**
3.7: #23-37 odd	**Variation**

possible rational zeros:

$$\frac{\pm \text{ factors of constant term}}{\text{factors of leading coefficient}}$$

$$f(x) = 8x^5 - 4x^3 + 2x - 9$$

$$\pm \frac{1, 3, 9}{1, 2, 4, 8} = \pm\left(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, 9, \frac{9}{2}, \frac{9}{4}, \frac{9}{8}\right)$$

$$f(x) = 8x^5 - 4x^3 + 2x$$

$$= x(\underbrace{8x^4 - 4x^2 + 2})$$

$$= 2x(4x^4 - 2x^2 + 1)$$

$$\frac{1}{1, 2, 4} \quad \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$$

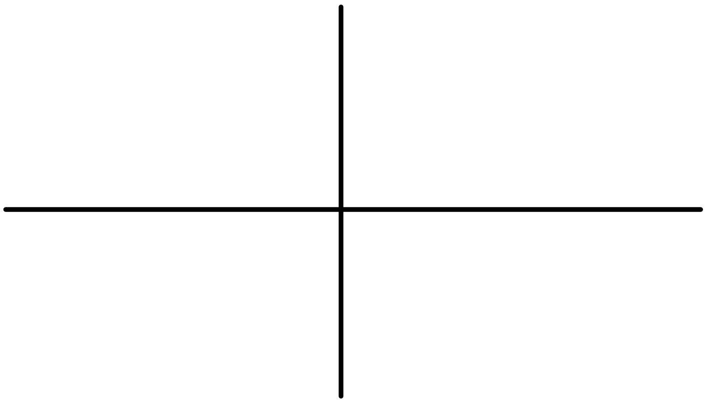
$$f(x) = \frac{(x-1)(x+3)}{x-2}$$

zero(s):

y-intercept:

vertical asymptote(s):

end behavior:



Limits:

as x -->	f(x) -->

$$f(x) = \frac{2x^2(x+3)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$

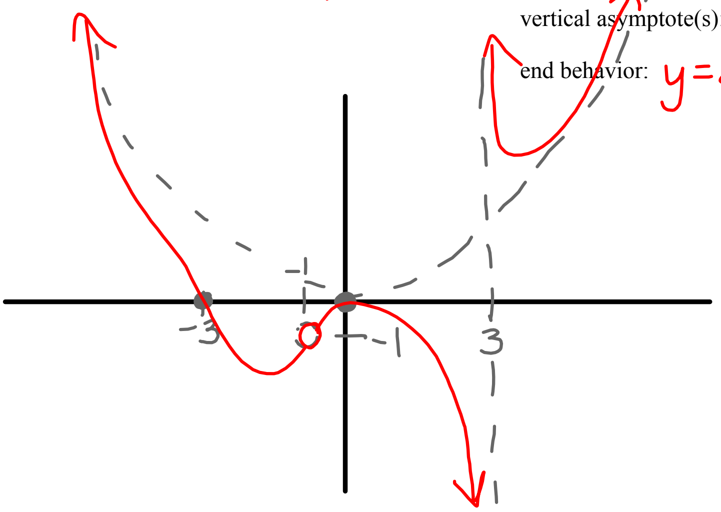
zero(s): $-3, 0$

y-intercept: $(0,0)$

vertical asymptote(s): $x=3$

end behavior: $y=2x^2$

hole @ $(-1,-1)$



Limits:

as x -->	f(x) -->

3.6 Polynomial and Rational Inequalities

lessthAND $\cap$ intersection
greatOR $\cup$ union

Solving basic inequalities review:

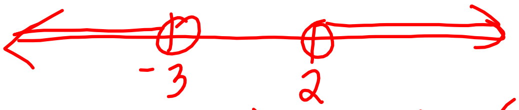
Linear:

$2x+1>5$
$2x>4$
$x>2$
$\{x \mid x>2\}$
$(2,\infty)$

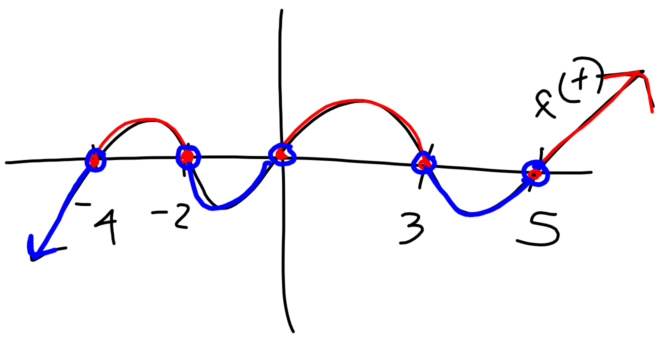
Absolute value:

$|2x+1|>5$

$2x+1>5$ or $2x+1<-5$
$x>2$ $2x<-6$
$x<-3$



$(-\infty,-3)\cup(2,\infty)=\{x \mid x<-3 \text{ or } x>2\}$



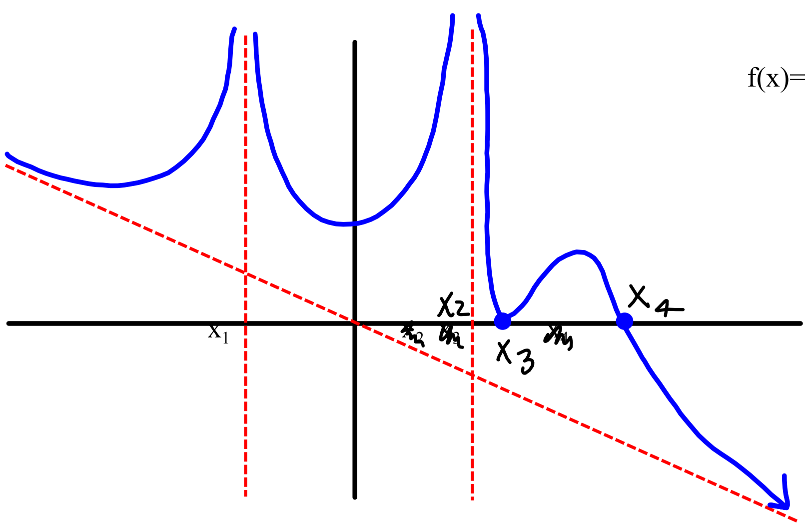
$f(x)\geq 0$ Where is f(x) positive or zero?
$[-4,-2]\cup[0,3]\cup[5,\infty)$

$f(x)<0$ Where is f(x) negative?
$(-\infty,-4)\cup(-2,0)\cup(3,5)$

$$\frac{5x^3+7x^2+3x-1}{2x+4} \geq \frac{3x^2+7}{2x^3-5x}$$

This inequality is hard to solve algebraically!
It's much easier to compare it to zero
and ask "where is it positive or negative?"

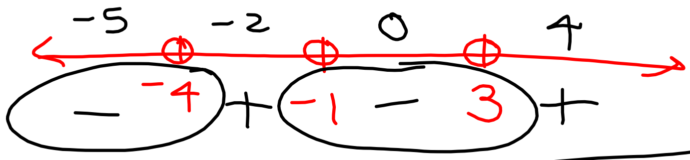
The *only* x-values at which the value of f(x) *can* change from positive to negative (or negative to positive) are at x-intercepts and vertical asymptotes (but it doesn't necessarily have to change at either).



$$f(x)= \frac{-(x-x_3)^2(x-x_4)}{(x-x_1)(x-x_2)}$$

$$(x+4)(x-3)(x+1) < 0$$

zeros: $-4,\ 3,\ -1$



$$(-\infty,-4)\cup(-1,3)$$

$$x^2+6x \geq 7$$

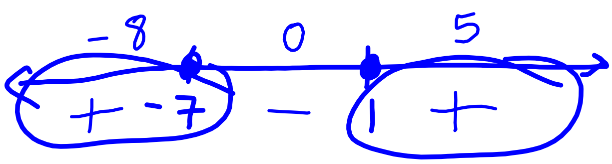
1. rearrange to compare to zero

$$x^2+6x-7 \geq 0$$

2. factor to find zeros (and/or vertical asymptotes)

$$(x+7)(x-1) \geq 0$$

3. split real number line into intervals according to values found in step 2; test a value in each interval to determine if the expression being compared to zero is positive or negative in that interval



$$(-\infty,-7]\cup[1,\infty)$$

$$\frac{(x+5)(x+1)(x-2)}{(x-3)(x+4)} \geq 0$$

$-6 \quad -4.5 \quad -2 \quad 0 \quad 2.5 \quad 4$

$- \quad -5 \quad + \quad -4 \quad - \quad -1 \quad + \quad 2 \quad - \quad 3 \quad +$

$[-5,-4) \cup [-1,2] \cup (3,\infty)$

$$\frac{(x-6)^2(x+3)}{x(x+4)} \leq 0 \qquad \boxed{(-\infty,-4) \cup [-3,0)}$$

$-5 \quad -3.5 \quad -1 \quad 1 \quad 7$

$- \quad -4 \quad + \quad -3 \quad - \quad 0 \quad + \quad 6 \quad +$

3.6

$\frac{1}{12} - \frac{1}{8}$

$\frac{2}{2} \cdot \frac{1}{3 \cdot 4} - \frac{1}{2 \cdot 4} \cdot \frac{3}{3}$

56. $\frac{3}{x^2-4} \leq \frac{5}{x^2+7x+10}$

$$\frac{3}{(x-2)(x+2)} \cdot \frac{x+5}{x+5} - \frac{5}{(x+5)(x+2)} \cdot \frac{x-2}{x-2} \leq 0$$

$$\frac{3(x+5)-5(x-2)}{(x-2)(x+2)(x+5)} \leq 0$$

$$\frac{3x+15-5x+10}{(\quad)(\quad)(\quad)} \leq 0$$

$$\frac{-2x+25}{(\quad)(\quad)(\quad)} \leq 0$$

$$\frac{-2\left(x-\frac{25}{2}\right)}{(x-2)(x+2)(x+5)} \leq 0$$

$-6 \quad -3 \quad 0 \quad 3 \quad 20$

$- \quad -5 \quad + \quad -2 \quad - \quad 2 \quad + \quad 12.5 \quad -$

$(-\infty, -5) \cup (-2, 2) \cup [12.5, \infty)$