3.6	#15-39odd	Solving polynomial inequalities
	#47, 53-61odd	Solving rational inequalities
3.7:	#23-37 odd	Variation
4.1	#17-23 odd	prove f is one-to-one; prove g is not one-to-one
	#59-63 odd	determine if f is on-to-one and if so, determine its inverse
	#77-81 odd	sketch the inverse function by reflecting over y=x
	#83-87 odd	use composition to show that the functions are inverses
4.2	#5-10all	match an exponential function to its graph
	#11-41odd	sketch graphs of exponential functions using transformations
	#43a,b,c,45,47	compound interest word problems
4.3	#1-8all	sketch graphs of logarithmic functions
	#9-33odd	evaluate log expressions without a calculator
	#35-53 odd	convert between logarithmic and exponential expressions
	#69-77 odd	apply change of base formula & calculator to approximate log expressions
	#83-90 all	graph logarithmic functions using transformations

4.1 - Inverse Functions

If a, then b
contrapositive:
If not b, then not a

Recall:

f is a function if each input value (x) has exactly one output f(x)

Functions pass the vertical line test.

f is a one-to-one function if, in addition, each y corresponds to only one x.

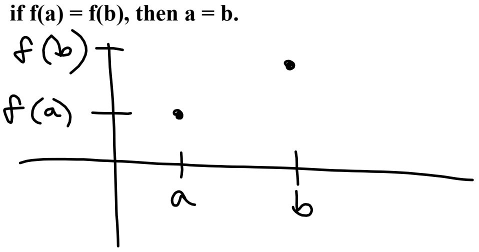
One-to-one functions pass both the horizontal line test and the vertical line test.

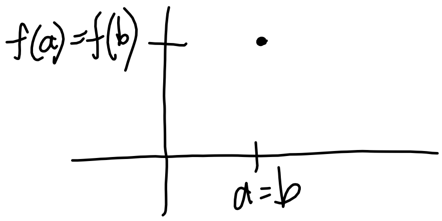
Formally, a function is one-to-one if different inputs have different outputs, i.e.,

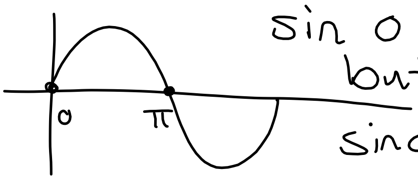
if a =/= b, then f(a) =/= f(b),

or equivalently, f is one-to-one if when the outputs are the same, the inputs are the same, i.e.,

if f(a) = f(b), then a = b.







$\sin 0 = \sin \pi$
but $0 \neq \pi$
since sin is not 1-1

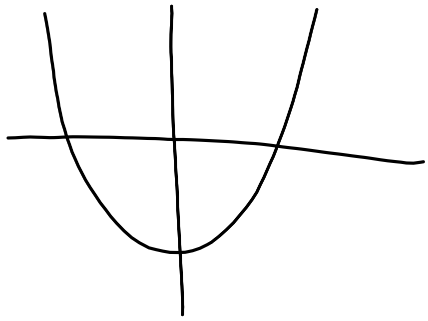
<u>Proving that a function *is* one-to-one</u> v. <u>proving that a function *is not* one-to-one</u>

(problems 17-24 from section 4.1)

If $a \neq b$, then $f(a) \neq f(b)$

To show that f(x) is <u>not</u> one-to-one, it is enough to provide a single counter-example, i.e. 2 different inputs that yield the same output

$f(x) = x^2 - 5$



$f(-5) = (-5)^2 - 5 = 20$

$f(5) = (5)^2 - 5 = 20$

since $f(-5) = f(5)$, f is <u>not</u> 1-1.

$f(x) = x^2 - 5$

$a^2 - 5 = b^2 - 5$

$\quad +5 \qquad +5$

$\sqrt{a^2} = \sqrt{b^2}$

$|a| = |b|$ ⟵ this does not prove it's not 1-1, it just fails to prove it is 1-1 (because it's not)

$\pm a = \pm b$

To show that f(x) <u>is</u> one-to-one, we must prove it in general.

If $f(a)=f(b)$, then $a=b$.

$$f(x) = -2x^3 + 1$$

$$f(a) = f(b)$$

$$-2(a)^3 + 1 = -2(b)^3 + 1$$

$$-1 \qquad\qquad -1$$

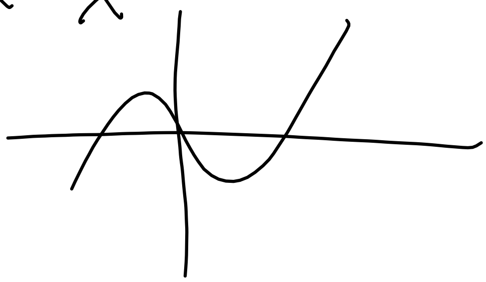
$$\frac{-2a^3}{-2} = \frac{-2b^3}{-2}$$

$$\sqrt[3]{a^3} = \sqrt[3]{b^3}$$

$$a = b$$

Since $f(a)=f(b)$ implies that $a=b$, f is indeed 1-1.

$$f(x) = x^2(x-1) = x^3 - x^2$$



$$\left.\begin{aligned} f(0) &= 0 \\ f(1) &= 0 \end{aligned}\right\} \text{not 1-1}$$

If a function is one-to-one, then it has an inverse.

Interchanging the first and second coordinates of each ordered pair in a relation produces the inverse function.

$f(x) = \{(1,2), (3,4), (5,6), (7,8)\}$

$f^{-1}(x) = \{(2,1), (4,3), (6,5), (8,7)\}$

If a relation is defined by an equation, interchanging the variables produces an equation of the inverse relation.

$y = -2x^3 + 1$

$x = -2y^3 + 1$

The domain of a one-to-one function f is the range of the inverse f^{-1}.

The range of a one-to-one function f is the domain of the inverse f^{-1}.

$f^{-1}(x) \neq \frac{1}{f(x)}$

Obtaining the formula for an inverse:

1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$

$f(x) = -2x^3 + 1$

$y = -2x^3 + 1$

$x = -2y^3 + 1$

$x - 1 = -2y^3$

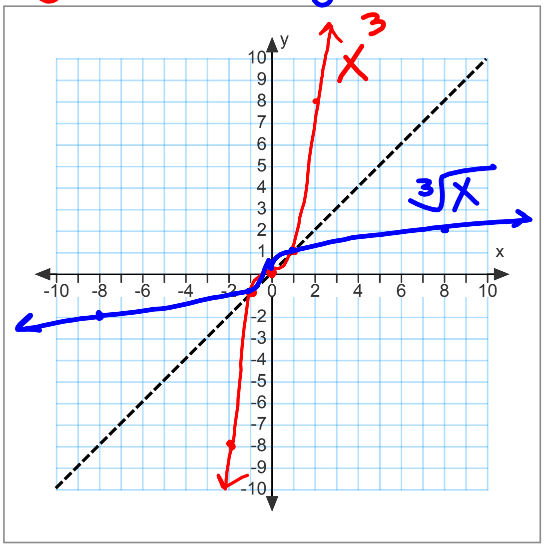
$\frac{x-1}{-2} = y^3$

$\sqrt[3]{\frac{x-1}{-2}} = y$

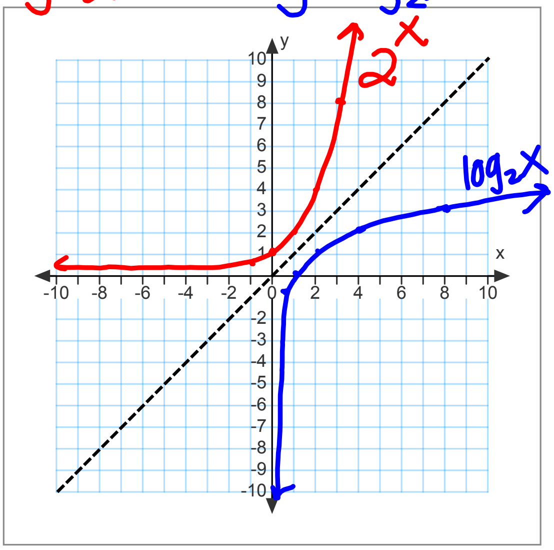
$f^{-1}(x) = \sqrt[3]{\frac{x-1}{-2}}$

The graph of f^{-1} is a reflection of the graph of f across the line $y = x$

$y = x^3$ $y = \sqrt[3]{x}$



$y = 2^x$ $y = \log_2 x$



If f & g are inverses, then

$(f \circ g)(x) = x$ for all x in the domain of g

AND

$(g \circ f)(x) = x$ for all x in the domain of f

86. $f(x) = \sqrt[3]{x+4}$; $f^{-1}(x) = x^3 - 4$

use composition to show f^{-1} is as given.

$$(f \circ f^{-1})(x) = \sqrt[3]{(x^3-4)+4} = \sqrt[3]{x^3} = x \checkmark$$

$$(f^{-1} \circ f)(x) = \left(\sqrt[3]{x+4}\right)^3 - 4 = x+4-4 = x \checkmark$$

88. $f(x) = \dfrac{x+6}{3x-4}$, $f^{-1}(x) = \dfrac{4x+6}{3x-1}$

$$f(f^{-1}(x)) = \frac{\frac{4x+6}{3x-1} + \frac{6}{1} \cdot \frac{3x-1}{3x-1}}{\frac{3}{1}\left(\frac{4x+6}{3x-1}\right) - \frac{4}{1} \cdot \frac{3x-1}{3x-1}}$$

$$= \frac{\left(\frac{4x+6+6(3x-1)}{3x-1}\right)}{\left(\frac{3(4x+6)-4(3x-1)}{3x-1}\right)}$$

$$= \frac{4x+6+18x-6}{\cancel{3x-1}} \cdot \frac{\cancel{3x-1}}{12x+18-12x+4}$$

$$= \frac{\cancel{22}x}{\cancel{22}} = x \checkmark$$

88. $f(x) = \dfrac{x+6}{3x-4}$, $f^{-1}(x) = \dfrac{4x+6}{3x-1}$

$$f^{-1}(f(x)) = \frac{\frac{4}{1}\left(\frac{x+6}{3x-4}\right) + \frac{6}{1}\cdot\frac{(3x-4)}{3x-4}}{\frac{3}{1}\left(\frac{x+6}{3x-4}\right) - 1\cdot\frac{(3x-4)}{3x-4}} =$$

$$= \frac{4x+24+18x-24}{3x+18-3x+4}$$

$$= \frac{22x}{22} = x \quad \checkmark$$