

- 3.6 #15-39 odd Solving polynomial inequalities
 #47, 53-61 odd Solving rational inequalities
- 3.7: #23-37 odd Variation
- 4.1 #17-23 odd prove f is one-to-one; prove g is not one-to-one
 #59-63 odd determine if f is on-to-one and if so, determine its inverse
 #77-81 odd sketch the inverse function by reflecting over $y=x$
 #83-87 odd use composition to show that the functions are inverses
- 4.2 #5-10 all match an exponential function to its graph
 #11-41 odd sketch graphs of exponential functions using transformations
 #43a,b,c,45,47 compound interest word problems
- 4.3 #1-8 all sketch graphs of logarithmic functions
 #9-33 odd evaluate log expressions without a calculator
 #35-53 odd convert between logarithmic and exponential expressions
 #69-77 odd apply change of base formula & calculator to approximate log expressions
 #83-90 all graph logarithmic functions using transformations

Review:

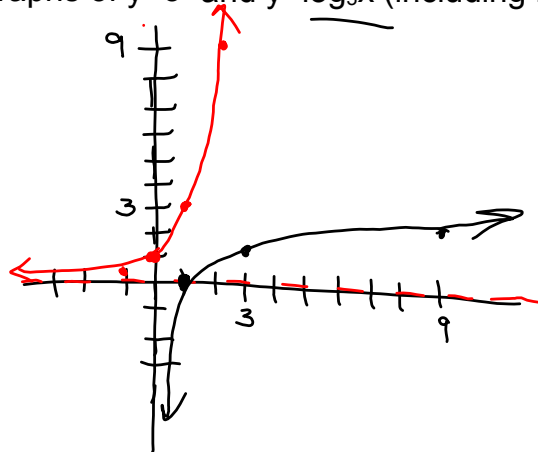
In the expression $\log_a b = c$, $\leftrightarrow a^c = b$
c is the power to which we raise a to get b.

Evaluate the following: $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$\log_2 16 = 4$ $\log_2(1/4) = -2$ $\log(100) = 2 = \log_{10} 100$

$\log(1/10) = -1$ $\ln(e) = 1$ $\ln(1) = 0$
log₁₀ to *log_e e* *log_e 1*

Sketch rough graphs of $y=3^x$ and $y=\log_3 x$ (including intercepts & asymptotes!)



x	$\log_3 x$
1	0 $(3^0 = 1)$
3	1 $3^1 = 3$
9	2 $3^2 = 9$

4.4 Properties of Logarithmic Functions

Review-Properties of Exponents

$$a^m a^n = a^{m+n}$$

$$\sqrt[n]{a} = a^{1/n}$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{a^m}{a^n} = \frac{a^{m-n}}{1} = \frac{1}{a^{n-m}}$$

Product Rule

$$\log_a(M \cdot N) =$$

$M, N > 0$

$$= \log_a M + \log_a N$$

Power Rule

$$\log_a(M^p)$$

$$= \log_a(\underbrace{M \cdot M \cdot M \cdots M}_{p \text{ times}})$$

$$= \underbrace{\log_a M + \log_a M + \cdots + \log_a M}_{p \text{ times}}$$

$$\log_a M^p = p \log_a M$$

Quotient Rule

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

common **errors**

$$\log_a MN \neq (\log_a M)(\log_a N)$$

$$\log_a (M+N) \neq \log_a M + \log_a N$$

$$\log_a \frac{M}{N} \neq \frac{\log_a M}{\log_a N} \quad \left. \vphantom{\log_a \frac{M}{N}} \right\} = \log_x M$$

$$(\log_a M)^p \neq p \log_a M = \log_a (M^p)$$

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Inverse Properties

$$\log_a a^x = x \quad ; \quad a^{\log_a x} = x$$

$$4^x = 5$$

$$\log_4 4^x = \log_4 5$$

$$x = \log_4 5$$

$$\log_2 x = 3$$

$$2^{\log_2 x} = 2^3$$

$$x = 2^3$$

Examples

$$2. \log_2(8 \cdot 64) = \log_2 8 + \log_2 64$$

$$= 3 + 6 = \boxed{9}$$

$$10. \log_a X^4 = 4 \log_a X$$

$$16. \ln \sqrt{a} = \ln a^{1/2} = \frac{1}{2} \ln a$$

$$22. \log_b \frac{3}{w} = \log_b 3 - \log_b w$$

$$\begin{aligned}
 32. \log_c \sqrt[3]{\frac{y^3 z^2}{x^4}} &= \log_c \left(\frac{y^3 z^2}{x^4} \right)^{1/3} \\
 &= \log_c \left(\frac{y z^{2/3}}{x^{4/3}} \right) = \log_c (y z^{2/3}) - \log_c (x^{4/3}) \\
 &= \log_c y + \log_c z^{2/3} - \log_c x^{4/3} \\
 &= \log_c y + \frac{2}{3} \log_c z - \frac{4}{3} \log_c x
 \end{aligned}$$

$$36. \log 0.01 + \log 1000$$

$$= \log(0.01 \cdot 1000)$$

$$= \log 10 = \boxed{1}$$

$$44. \ln 2x + 3(\ln x - \ln y)$$

$$= \ln 2x + 3 \ln \frac{x}{y}$$

$$= \ln 2x + \ln \left(\frac{x}{y}\right)^3$$

$$= \ln \left(2x \left(\frac{x}{y}\right)^3\right) = \boxed{\ln \frac{2x^4}{y^3}}$$

$$50. \frac{2}{3} [\ln(x^2-9) - \ln(x+3)] + \ln(x+y)$$

rewrite as a single logarithm

$$= \frac{2}{3} \ln \frac{x^2-9}{x+3} + \ln(x+y)$$

$$= \frac{2}{3} \ln \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} + \ln(x+y)$$

$$= \frac{2}{3} \ln (x-3)^2 + \ln(x+y)$$

$$= \ln (x-3)^{2/3} + \ln(x+y)$$

$$= \ln [(x-3)^{2/3} (x+y)]$$

$$\log_a 2 \approx 0.301, \log_a 7 \approx 0.845, \log_a 1 \approx 1.041$$

$$54. \log_a 14 = \log_a (2 \cdot 7) \\ = \log_a 2 + \log_a 7 \\ = 0.301 + 0.845 = \boxed{1.146}$$

$$56. \log_a \frac{1}{7} = \log_a 1 - \log_a 7 = 0 - 0.845 = \boxed{-0.845}$$

$$\log_a 7^{-1} = -\log_a 7$$

$$58. \log_a 9 = \cancel{\log_a (11-2)} = \cancel{\log_a (7+2)}$$