4.5	#1-25 odd; #27-47 odd	solving exponential equations; solving logarithmic equations
4.6	#5,7,9,15,17	application problems with logs/exp functions
10.1	#7,9,23-31odd #59,63,67	find the general (nth) term of the sequence write sigma notation
10.2	#9,15,19,21,25,29, 35,37	arithmetic sequences and series
10.3	#15,19,21,25,35,37,43,45,49,57	geometric sequences and series
10.7	#1,7,21,27,31-39odd	binomial theorem

Quiz on log/exp equations & applications - next week

Test #4 - Feb 6-8??

10. In 1984, the average cell phone price was $3395, and in 2002, it was $145. Assuming the average price of a cell phone decreased according to the exponential model,

a. Find the value of k, and write an exponential function that describes the average price of a cell phone after time t, in years, where t is the number of years since 1984.

$1984: t=0 \quad P_0 = 3395$

$2002: t=18 \quad P(18) = 145$

$P(t) = P_0 e^{kt}$

$145 = 3395 e^{k \cdot 18}$

$\frac{145}{3395} = e^{k \cdot 18}$

$\ln\left(\frac{145}{3395}\right) = k \cdot 18$

$\frac{\ln\left(\frac{145}{3395}\right)}{18} = k$

$$\boxed{P(t) = 3395 e^{\frac{1}{18}\ln\left(\frac{145}{3395}\right) t}}$$

b. Estimate the price of a cell phone in 2017 ~~2006 and 2008 to the nearest dollar~~.

$2017: t = 33$

$P(33) = 3395 e^{\frac{1}{18}\ln\left(\frac{145}{3395}\right)(33)} \approx \boxed{\$10.47}$

c. At this decay rate, in what year will the price be $39?

$39 = 3395 e^{\frac{1}{18}\ln\left(\frac{145}{3395}\right) t}$

$\frac{39}{3395} = e^{\frac{1}{18}\ln\frac{145}{3395} t}$

$\ln\frac{39}{3395} = \frac{1}{18}\ln\frac{145}{3395} t$

$t = \frac{\ln\left(\frac{39}{3395}\right)}{\frac{1}{18}\ln\left(\frac{145}{3395}\right)}$

≈ 25

$\boxed{2009}$

10.1 Sequences and Series

An **infinite sequence** is a function that has as its domain the set of natural numbers.

$$\begin{array}{ccccccccccccc} 1 & , & 2 & , & 3 & , & 4 & , & 5 & , & \ldots & , & n & , & \ldots \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & & & & & \\ 2 & , & 4 & , & 8 & , & 16 & , & 32 & , & \ldots & , & & , & \ldots \\ 2^1 & & 2^2 & & 2^3 & & 2^4 & & 2^5 & & & & 2^n & & \end{array}$$

What is the n^{th} term?

A **finite sequence** has as its domain {1, 2, 3, ... , n} for some n.

A **series** is a sum of terms of a sequence.

$$a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{n} a_i$$

This notation is called "sigma notation" and is read as "the sum from i equals 1 to n of a sub i."

10.1

2. $a_n = (n-1)(n-2)(n-3)$

$a_1 = (1-1)(1-2)(1-3) = 0$

$a_2 = (2-1)(2-2)(2-3) = 0$

$a_3 = (3-1)(3-2)(3-3) = 0$

$a_4 = (4-1)(4-2)(4-3) = 3 \cdot 2 \cdot 1 = 6$

$a_5 = (5-1)(5-2)(5-3) = 4 \cdot 3 \cdot 2 = 24$

8. $a_n = (-1)^{n-1}(3n-5)$

alternating sequence

$(-1)^n$

$a_1 = (-1)^{1-1}(3\cdot 1-5) = \boxed{-2}$

$a_2 = (-1)^{2-1}(3\cdot 2-5) = \boxed{-1}$

$a_3 = (-1)^{3-1}(3\cdot 3-5) = \boxed{4}$

$a_4 = (-1)^{4-1}(3\cdot 4-5) = \boxed{-7}$

$a_5 = (-1)^{5-1}(3\cdot 5-5) = \boxed{10}$

what is the n^{th} term?

26. $-2, 3, 8, 13, 18, \ldots$

$\boxed{a_n = 5n-7}$

$5, 10, 15, 20, 25$

$5\cdot 1 \quad 5\cdot 2 \quad 5\cdot 3 \quad 5\cdot 4 \quad 5\cdot 5$

$a_n = 5n$

32. $\ln e^2, \ln e^3, \ln e^4, \ln e^5, \ldots = 2, 3, 4, 5, \ldots$

$a_n = \ln e^{n+1} = n+1$

$$2+4+8+16+32$$
$$2^1+2^2+2^3+2^4+2^5=\sum_{n=1}^{5}2^n$$

$$2^1+2^2+2^3+\dots+2^n=\sum_{i=1}^{n}2^i$$

$$2^1+2^2+2^3+\dots=\sum_{k=1}^{\infty}2^k$$

$$2^0+2^1+\dots+2^{5000}=\sum_{n=0}^{5000}2^n$$

$$2^{13}+2^{14}+\dots+2^{19}+2^{52}+2^{53}+\dots+2^{99}$$
$$=\sum_{i=13}^{19}2^i+\sum_{i=52}^{99}2^i$$

10.1 Sequences and Series

write sigma notation. $\sum_{i=1}^{n}a_n$

56. $7+14+21+28+35+\dots$

$7\cdot1+7\cdot2+7\cdot3+7\cdot4+7\cdot5+\dots\ 7\cdot n$

$$=\boxed{\sum_{n=1}^{\infty}7n}$$

60. $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}$

$= \boxed{\sum_{n=1}^{5} \frac{1}{n^2}} \qquad = \sum_{n=1}^{5} n^{-2}$

62. $9-16+25+\ldots+(-1)^{n+1} n^2$

$= \sum_{i=3}^{n} (-1)^{i+1} i^2$

$-9+16-25+\ldots+(-1)^n \cdot n^2$

$= \sum_{i=3}^{n} (-1)^{i} i^2$

64.

$$\frac{1}{1\cdot 2^2}+\frac{1}{2\cdot 3^2}+\frac{1}{3\cdot 4^2}+\frac{1}{4\cdot 5^2}+\ldots$$

$$=\boxed{\sum_{n=1}^{\infty}\frac{1}{n(n+1)^2}}$$

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

1 2 3 4 5 6 7 8 9, ...

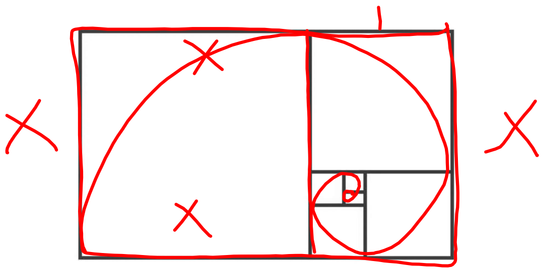
Recursively-defined sequence defines an element of the sequence interms of other elements.

$$F_n = F_{n-1} + F_{n-2}$$

$$a_n = \left(1 + \frac{1}{n}\right)^n \to e$$

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \ldots$$

$\to$ $1.618 \approx \varphi$ "The Golden Ratio"



The Golden Rectangle

$$\frac{x+1}{x} = \frac{x}{1}$$

$$x = \frac{1+\sqrt{5}}{2}$$

$$= \varphi$$