

18. A certain element has a half-life of 30 years. If we start with a 5000 gram sample of the element, after how many years will there be only 100 grams left?

a. Find the exponential decay constant, k . ~~Leave the answer in terms of the natural log.~~

$$K = \frac{-\ln 2}{30} \approx -0.0231$$

b. State the exponential decay function for the amount of substance left after time t , with correct values for P_0 and k .

$$P(t) = 5000e^{-0.0231t}$$

c. Determine the number of years t it will take for there to be only 100 grams of the element left.

$$100 = 5000e^{-0.0231t}$$

$$\frac{100}{5000} = e^{-0.0231t}$$

$$\ln \frac{1}{50} = -0.0231t$$

$$\frac{+\ln 50}{+0.0231} = t \approx \boxed{169 \text{ years}}$$

4.5 #1-25 odd;
#27-47 odd

solving exponential equations;
solving logarithmic equations

4.6 #5,7,9,15,17

application problems with logs/exp functions

10.1 #7,9,23-31 odd
#59,63,67

find the general (nth) term of the sequence
write sigma notation

10.2 #9,15,19,21,25,29, 35,37

arithmetic sequences and series

10.3 #15,19,21,25,35,37,43,45,49,57

geometric sequences and series

10.7 #1,7,21,27,31-39 odd

binomial theorem

Quiz on log/exp equations & applications - 8th per Wed. Feb 1; 1st per Thurs Feb 2

Test #4 - Mon. Feb 6

Arithmetic Sequences/Series:10.2

Definition: A sequence is arithmetic if there exists a number d , called the common difference, such that $a_{n+1} = a_n + d$ for any integer $n \geq 1$.

The nth term of an arithmetic sequence is given by

$$a_n = a_1 + (n - 1)d, \text{ for any integer } n \geq 1.$$

The sum of the first n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n) = \sum_{i=1}^n a_i$$

Geometric Sequences/Series:10.3

Definition: A sequence is geometric if there is a number r , called the common ratio, such that

$$\frac{a_{n+1}}{a_n} = r, \text{ or } a_{n+1} = a_n r, \text{ for any integer } n \geq 1.$$

The nth term of a geometric sequence is given by

$$a_n = a_1 r^{n-1}, \text{ for any integer } n \geq 1.$$

The sum of the first n terms of a geometric sequence is given by

$$S_n = \frac{a_1(1-r^n)}{1-r}, \text{ for any } r \neq 1.$$

When $|r| < 1$, the limit or sum of an infinite geometric series is given by

$$S_\infty = \frac{a_1}{1-r}$$

Find Fraction Notation.

$$52. \quad 0.222\dots = 0.\overline{2} = 0.2 + 0.02 + 0.002 + 0.0002 + \dots$$

$$\begin{array}{r} 10x = 2.\overline{2} \\ - x = 0.\overline{2} \\ \hline 9x = 2 \end{array} \qquad = \frac{0.2}{1-0.1} = \frac{0.2}{0.9} \cdot \frac{10}{10} = \boxed{\frac{2}{9}}$$

$$\boxed{x = \frac{2}{9}}$$

$$54. \quad \begin{array}{r} 6.1\overline{61616} \\ 6.1 \\ + 0.061 \\ + 0.00061 \\ + 0.0000061 \end{array} = \frac{6.1}{1-0.01} = \frac{6.1}{0.99} \cdot \frac{100}{100} = \boxed{\frac{610}{99}}$$

10.3 #58

Someone offers you a job for the month of February (28 days). You will be paid \$0.01 on the 1st day, \$0.02 on the 2nd day, \$0.04 on the 3rd day, doubling your previous day's salary each day. Do you take the job? *Yes, please!*

$$0.01 + 0.02 + 0.04 + \dots$$

$$\sum_{n=0}^{27} (0.01)2^n = \frac{0.01(1-2^{28})}{1-2}$$

$$\approx \$2,684,354.55$$

10.7 The Binomial TheoremExpansion of $(a + b)^n$

$(a + b)^0 = 1$

$(a + b)^1 = a + b$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

What patterns do you see?

 $(a+b)^n$ has $n+1$ terms

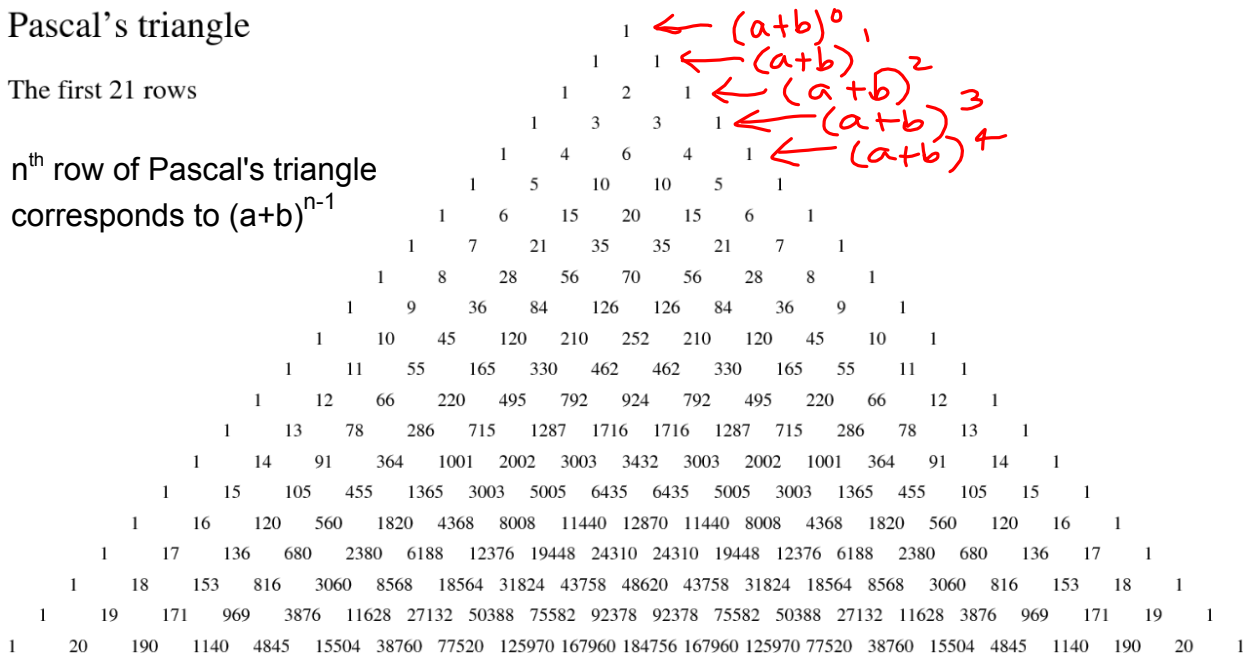
exp. on a decreases from n to 0
 exp on b increases from 0 to n
 sum of exponents on any term is n

 a^n & b^n have coeff 1 $a^{n-1}b$ & ab^{n-1} have coeff n

Pascal's triangle

The first 21 rows

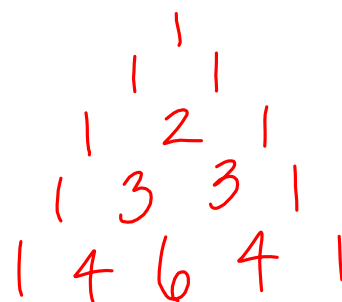
n^{th} row of Pascal's triangle corresponds to $(a+b)^{n-1}$



Expand $(x-1)^4$ $(a+b)^n$
 $a=x, b=-1, n=4$

$$1x^4(-1)^0 + 4x^3(-1)^1 + 6x^2(-1)^2 + 4x^1(-1)^3 + 1x^0(-1)^4$$

$$= x^4 - 4x^3 + 6x^2 - 4x + 1$$



$$8. (2x-3y)^5 \quad (a+b)^n$$

$$a = 2x, b = -3y, n = 5$$

$$\begin{array}{cccccc} & & & & & 1 \\ & & & & & 1 & 1 \\ & & & & & 1 & 2 & 1 \\ & & & & & 1 & 3 & 3 & 1 \\ & & & & & 1 & 4 & 6 & 4 & 1 \\ & & & & & 1 & 5 & 10 & 10 & 5 & 1 \end{array}$$

$$\begin{aligned} & \frac{1 \binom{5}{0} (2x)^5 (-3y)^0 + 5 \binom{5}{1} (2x)^4 (-3y)^1 + 10 \binom{5}{2} (2x)^3 (-3y)^2 + 10 \binom{5}{3} (2x)^2 (-3y)^3 + 5 \binom{5}{4} (2x)^1 (-3y)^4 + 1 \binom{5}{5} (2x)^0 (-3y)^5}{=} \\ & = 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5 \end{aligned}$$

$$(2x-y)^5 = ?$$

$$\begin{aligned} & = 1 \binom{5}{0} (2x)^5 (-y)^0 + 5 \binom{5}{1} (2x)^4 (-y)^1 + 10 \binom{5}{2} (2x)^3 (-y)^2 + 10 \binom{5}{3} (2x)^2 (-y)^3 + 5 \binom{5}{4} (2x)^1 (-y)^4 + 1 \binom{5}{5} (2x)^0 (-y)^5 \\ & = 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5 \end{aligned}$$

Binomial Coefficients

$$\binom{n}{k} = \text{"n choose k"}$$

= the total # of combinations
of n objects taken
 k at a time

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

"n factorial"
 $n! = n(n-1)(n-2)\dots 2 \cdot 1$
 $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Given a class of 9 students,
how many groups of 2 can we make?

$$\binom{9}{2} = \frac{9!}{2!(9-2)!} = \frac{9 \cdot 8 \cdot \cancel{7!}}{2 \cdot 1 \cdot \cancel{7!}} = \boxed{36}$$

9C2