18. A certain element has a half-life of 30 years. If we start with a 5000 gram sample of the element, after how many years will there be only 100 grams left?

a. Find the exponential decay constant, k. ~~Leave the answer in terms of the natural log.~~

$$K = \frac{-\ln 2}{30} \approx -0.0231$$

b. State the exponential decay function for the amount of substance left after time t, with correct values for P_0 and k.

$$P(t) = 5000e^{-0.0231t}$$

c. Determine the number of years t it will take for there to be only 100 grams of the element left.

$$100 = 5000e^{-0.0231t}$$

$$\frac{1\cancel{00}}{50\cancel{00}} = e^{-0.0231t}$$

$$\ln\frac{1}{50} = -0.0231t$$

$$\frac{+\ln 50}{+0.0231} = t \approx \boxed{169 \text{ years}}$$

4.5 #1-25 odd; #27-47 odd	solving exponential equations; solving logarithmic equations
4.6 #5,7,9,15,17	application problems with logs/exp functions
10.1 #7,9,23-31odd #59,63,67	find the general (nth) term of the sequence write sigma notation
10.2 #9,15,19,21,25,29, 35,37	arithmetic sequences and series
10.3 #15,19,21,25,35,37,43,45,49,57	geometric sequences and series
10.7 #1,7,21,27,31-39odd	binomial theorem

Quiz on log/exp equations & applications - 8th per Wed. Feb 1; 1st per Thurs Feb 2

Test #4 - Mon. Feb 6

Arithmetic Sequences/Series: 10.2

Definition: A sequence is arithmetic if there exists a number d, called the **common difference**, such that $\boldsymbol{a_{n+1} = a_n + d}$ for any integer $n \geq 1$.

The **nth term** of an arithmetic sequence is given by
$\boldsymbol{a_n = a_1 + (n-1)d}$, for any integer $n \geq 1$.

The **sum of the first n terms** of an arithmetic sequence is given by
$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)} = \sum_{i=1}^{n} a_i$

Geometric Sequences/Series: $\boxed{10.3}$

Definition: A sequence is geometric is there is a number r, called the **common ratio**, such that $\boldsymbol{\frac{a_{n+1}}{a_n} = r}$, or $a_{n+1} = a_n r$, for any integer $n \geq 1$.

The **nth term of a geometric sequence** is given by
$\boldsymbol{a_n = a_1 r^{n-1}}$, for any integer $n \geq 1$.

The **sum of the first n terms** of a geometric sequence is given by
$\boldsymbol{S_n = \frac{a_1(1-r^n)}{1-r}}$, for any $r \neq 1$.

When $|r| < 1$, the **limit or sum of an infinite geometric series** is given by
$$\boldsymbol{S_\infty = \frac{a_1}{1-r}}$$

Find Fraction Notation.

52. $0.222\ldots = 0.\overline{2} = 0.2 + 0.02 + 0.002 + 0.0002 + \ldots$

$$10x = 2.\overline{2}$$
$$-\ x = 0.\overline{2}$$
$$9x = 2$$
$$\boxed{x = \frac{2}{9}}$$

$$= \frac{0.2}{1-0.1} = \frac{0.2}{0.9} \cdot \frac{10}{10} = \boxed{\frac{2}{9}}$$

54. $6.16161\overline{6}$

$$= \frac{6.1}{1-0.01} = \frac{6.1}{0.99} \cdot \frac{100}{100}$$

$$6.1$$
$$+\ 0.061$$
$$+\ 0.00061$$
$$+\ 0.0000061$$

$$= \boxed{\frac{610}{99}}$$

10.3 #58

Someone offers you a job for the month of February (28 days). You will be paid \$0.01 on the 1st day, \$0.02 on the 2nd day, \$0.04 on the 3rd day, doubling your previous day's salary each day. Do you take the job? Yes, please!

$0.01 + 0.02 + 0.04 + \ldots$

$$\sum_{n=0}^{27} (0.01)2^n = \frac{0.01(1-2^{28})}{1-2}$$

$\approx \boxed{\$2{,}684{,}354.55}$

10.7 The Binomial Theorem

$(a+b)^n$ has $n+1$ terms

Expansion of $(a+b)^n$

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

What patterns do you see?

exp. on a decreases from n to 0

exp on b increases from 0 to n

sum of exponents on any term is n

a^n & b^n have coeff 1

$a^{n-1}b$ & ab^{n-1} have coeff n

Pascal's triangle

The first 21 rows

n^{th} row of Pascal's triangle corresponds to $(a+b)^{n-1}$

1 ← $(a+b)^0$
1 1 ← $(a+b)^1$
1 2 1 ← $(a+b)^2$
1 3 3 1 ← $(a+b)^3$
1 4 6 4 1 ← $(a+b)^4$
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1 19 171 969 3876 11628 27132 50388 75582 92378 92378 75582 50388 27132 11628 3876 969 171 19 1
1 20 190 1140 4845 15504 38760 77520 125970 167960 184756 167960 125970 77520 38760 15504 4845 1140 190 20 1

Expand $(x-1)^4$ $(a+b)^n$

$a=x,\ b=-1,\ n=4$

$$\underline{1x^4(-1)^0} + \underline{4x^3(-1)^1} + \underline{6x^2(-1)^2} + \underline{4x^1(-1)^3} + \underline{1x^0(-1)^4}$$

$$= \boxed{x^4 - 4x^3 + 6x^2 - 4x + 1}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

8. $(2x-3y)^5$ $\quad (a+b)^n \quad a=2x,\ b=-3y,\ n=5$

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

$$1(2x)^5(-3y)^0+5(2x)^4(-3y)^1+10(2x)^3(-3y)^2+10(2x)^2(-3y)^3+5(2x)^1(-3y)^4+1(2x)^0(-3y)^5$$

$$=\boxed{32x^5-240x^4y+720x^3y^2-1080x^2y^3+810xy^4-243y^5}$$

$(2x-y)^5=?$

$$=1(2x)^5(-y)^0+5(2x)^4(-y)^1+10(2x)^3(-y)^2+10(2x)^2(-y)^3+5(2x)^1(-y)^4+1(2x)^0(-y)^5$$

$$=\boxed{32x^5-80x^4y+80x^3y^2-40x^2y^3+10xy^4-y^5}$$

Binomial Coefficients

$\binom{n}{k}$ = "n choose k"

= the total # of combinations of n objects taken k at a time

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

"n factorial"

$n! = n(n-1)(n-2)\cdots 2\cdot 1$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

Given a class of 9 students, how many groups of 2 can we make?

$$\binom{9}{2} = \frac{9!}{2!(9-2)!} = \frac{9\cdot 8\cdot \cancel{7!}}{2\cdot 1\cdot \cancel{7!}} = \boxed{36}$$

9C2