4.5	#1-25 odd; #27-47 odd	solving exponential equations; solving logarithmic equations
4.6	#5,7,9,15,17	application problems with logs/exp functions
10.1	#7,9,23-31odd #59,63,67	find the general (nth) term of the sequence write sigma notation
10.2	#9,15,19,21,25,29, 35,37	arithmetic sequences and series
10.3	#15,19,21,25,35,37,43,45,49,57	geometric sequences and series
10.7	#1,7,21,27,31-39odd	binomial theorem

Test #4 - Mon. Feb 6

Final Exam - Wed. Feb 15

1. c $\log x =$

a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

2. b $\log_a b =$

a. $\frac{\log_b x}{\log_a x}$ b. $\frac{\log b}{\log a}$ c. $b \log a$ d. $\frac{\log a}{\log b}$

3. b $\log_a(MN) =$

a. $\log_a M \log_a N$ b. $\log_a M + \log_a N$ c. $\log_a(M+N)$ d. $N \log_a M$

4. c $\log_a 1 =$

a. a b. 1 c. 0 d. e^a

5. a $\ln x =$

a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

6. b $\log_a M^p =$

a. $(\log_a M)^P$ b. $P \log_a M$ c. $(\log a)(M^p)$ d. $MP \log a$

7. d $\log_a \left(\frac{M}{N}\right) =$

a. $\frac{\log_a M}{\log_a N}$ b. $\frac{\ln M}{\ln a}$ c. $\log_a(M - N)$ d. $\log_a M - \log_a N$

8. b $\log_a a =$

a. a b. 1 c. 0 d. e^a

9. d

Convert to an exponential equation: $\log_5 x = 10$.

a) $x = 10^5$ b) $x^{10} = 5$ c) $10^x = 5$ d) $x = 5^{10}$

$5^{10} = x$

$\log_a b = c \Leftrightarrow a^c = b$

10. a

Convert to a logarithmic equation: $4^x = 12.8$.

a) $x = \log_4 12.8$ b) $x = \log_{12.8} 4$

c) $\log_x 12.8 = 4$ d) $\log_x 4 = 12.8$

$\log_4(4^x) = \log_4(12.8)$

$x = \log_4(12.8)$

11. d

Solve: $\log_{100} 10 = x$.

a) $\frac{1}{10}$ b) 2 c) -2 d) $\frac{1}{2}$

$\log_{100} 10 = \frac{\log_{10} 10}{\log_{10} 100} = \frac{1}{2}$

12. c

Solve: $\log(\ln x) = 0$.

a) 0 b) 1 c) e d) 10^e

$\log_{10}(\log_e x) = 0$

$10^0 = \log_e x$

$1 = \log_e x$

$e^1 = x$

13. b

Solve: $2^{5+x} = 32^{x-4}$

a) $\frac{69}{15}$ b) $\frac{25}{4}$ c) $\frac{9}{4}$ d) $\frac{15}{4}$

$(a^m)^n = a^{mn}$

$2^{5+x} = (2^5)^{x-4}$

$2^{5+x} = 2^{5(x-4)}$

$\log_2(2^{5+x}) = \log_2(2^{5(x-4)})$

$5+x = 5x-20$

$25 = 4x \longrightarrow x = \frac{25}{4}$

14. 8.66%

The population of a country doubled in 8 yr. What was the exponential growth rate?

$P(8) = 2P_0$

$P(t) = P_0 e^{kt}$

$\frac{2P_0}{P_0} = \frac{P_0 e^{k\cdot 8}}{P_0}$

$2 = e^{k\cdot 8}$

$\ln 2 = k\cdot 8$

$\frac{\ln 2}{8} = k$

xx.xx%

15. a

Solve: $\log_3(3x+6) - \log_3(x-6) = 2$.

a) 10 b) $-\frac{3}{2}$ c) 14 d) Does not exist

$\log_a b = c \Leftrightarrow a^c = b$

$\log_3 \frac{3x+6}{x-6} = 2$

$3^{\log_3 \frac{3x+6}{x-6}} = 3^2$

$\frac{3x+6}{x-6} = 9$

$3x+6 = 9(x-6)$

$3x+6 = 9x-54$

$60 = 6x$

$10 = x$

$P_0 = 8000$ $P(t) = P_0 e^{kt}$

16. a. 6.00%

Suppose \$8000 is invested at interest rate k, compounded continuously, and grows to \$11,466.64 in 6 years. ← $P(6) = 11466.64$

b. $P(t) = 8000e^{0.06t}$

a. Find the interest rate, as a percentage, to the nearest hundredth.**

***Use this rounded answer in answering all remaining questions.*

b. State the exponential growth function for this particular investment with appropriate variables and constants.

c. Find the balance after 10 years.

d. Find the doubling time.

c. \$14,576.95

d. 11.55yrs

$11466.64 = 8000e^{k\cdot 6}$

$\frac{11466.64}{8000} = e^{k\cdot 6}$

$\ln\frac{11466.64}{8000} = k\cdot 6$

$\frac{\ln\left(\frac{11466.64}{8000}\right)}{6} = k$

$P(10) = 8000e^{0.06(10)}$

$\ln 2 = k \cdot T$

$\frac{\ln 2}{0.06} = T$

These are the only formulas you will be given on the ~~final~~ Test 4

Make sure you know any and all other formulas (logs, exponents, parabola vertex, etc.) AND make sure you know what the variables in these mean and when to use which.

$$\boldsymbol{a_{n+1} = a_n + d}$$

$$\boldsymbol{a_n = a_1 + (n-1)d}$$

$$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$$

$$\boldsymbol{a_n = a_1 r^{n-1}}$$

$$\boldsymbol{S_n = \frac{a_1(1-r^n)}{1-r}}$$

$$\boldsymbol{S_\infty = \frac{a_1}{1-r}}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$P(t) = P_0 e^{kt}$$

1. Find the 32nd term of the arithmetic sequence $92, 87, 82, 77, 72, \ldots$

$a_n = a_1 + (n-1)d$

$a_{32} = 92 + (32-1)(-5) = \boxed{-63}$

2. Determine the sum of the first 19 terms of the geometric series $-81 + 27 - 9 + 3 - 1 + \cdots$

$S_{19} = \dfrac{-81\left(1-\left(-\frac{1}{3}\right)^{19}\right)}{1-\left(-\frac{1}{3}\right)} \approx \boxed{-60.75}$ $S_n = \dfrac{a_1(1-r^n)}{1-r}$

3. Find and evaluate the sum.

$\sum_{k=1}^{\infty}\left(\frac{3}{5}\right)^k = \left(\frac{3}{5}\right)^1 + \left(\frac{3}{5}\right)^2 + \ldots$ $S_\infty = \dfrac{a_1}{1-r}$

$= \dfrac{3/5}{1-3/5} = \dfrac{3/5}{2/5} = \boxed{\dfrac{3}{2}}$

4. Write sigma notation for the series. $4 - 9 + 16 - 25 + \cdots + (-1)^n n^2$

$\sum_{k=2}^{n} (-1)^k k^2$

5. Write the 4th term of $(2x - y)^6$.

$(k+1)^{st}$ term of $(a+b)^n$ is $\binom{n}{k} a^{n-k} b^k$

$4 = k+1$

$a = 2x$; $b = -y$

$n = 6$; $k = 3$

$\binom{6}{3}(2x)^{6-3}(-y)^3 = \dfrac{6!}{3!(6-3)!}(2x)^3(-y^3)$

$= \dfrac{\cancel{6}\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3\cdot 2\cdot 1}\cdot \cancel{3!}}(8x^3)(-y^3) = 5\cdot 4\cdot 8\cdot x^3(-y^3)$

$= \boxed{-160x^3y^3}$

Binomial Coefficients

$\binom{n}{k}$ = "n choose k"

= the total # of combinations of n objects taken k at a time

$\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

"n factorial"

$n! = n(n-1)(n-2)\cdots 2\cdot 1$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

In particular,
The $(k+1)^{st}$ term of $(a+b)^n$ is

$$\binom{n}{k} a^{n-k} b^k$$

Given a set with n objects, the # of subsets containing k elements is $\binom{n}{k}$, so the total # of subsets of any size is $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$

$$(1+1)^n = \binom{n}{0} 1^n \cdot 1^0 + \binom{n}{1} 1^{n-1} \cdot 1^1 + \binom{n}{2} \cdot 1^{n-2} \cdot 1^2 + \cdots + \binom{n}{n} \cdot 1^0 \cdot 1^n$$

$$= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$$

$\Rightarrow$ The total number of ways to make a subset of any size (from 0 to n) from a set of n objects is 2^n

How many "words" (including the non-word, with no letters repeating, and order doesn't matter so that "no" is the same work as "on") **can we write with the English alphabet, which has 26 letters?**

$2^{26} = 67{,}108{,}864$ words