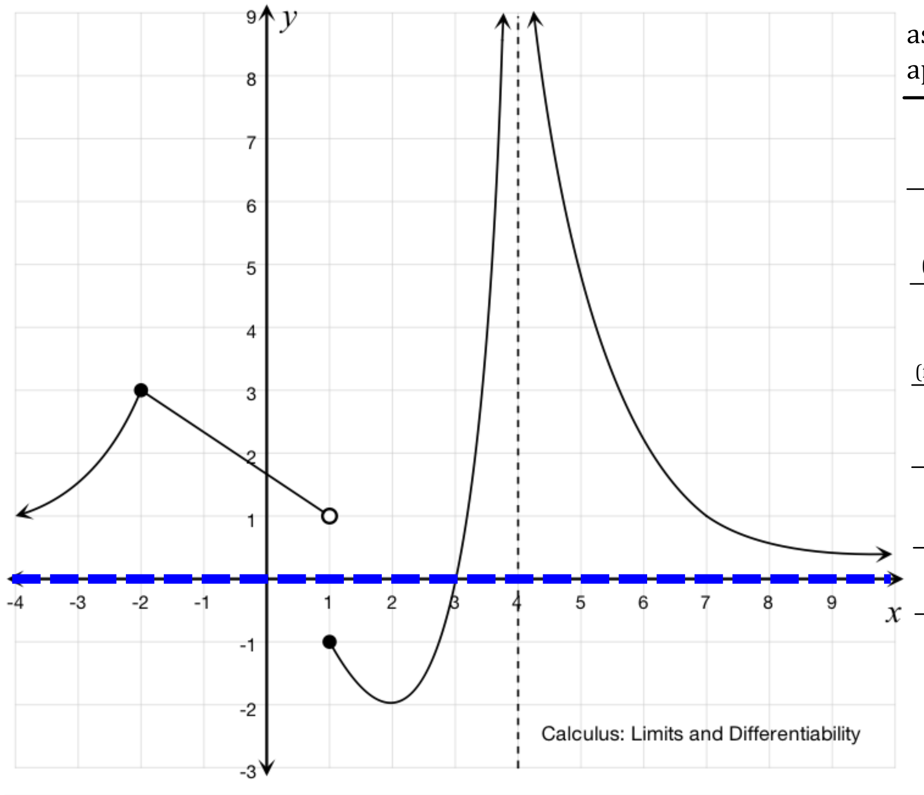
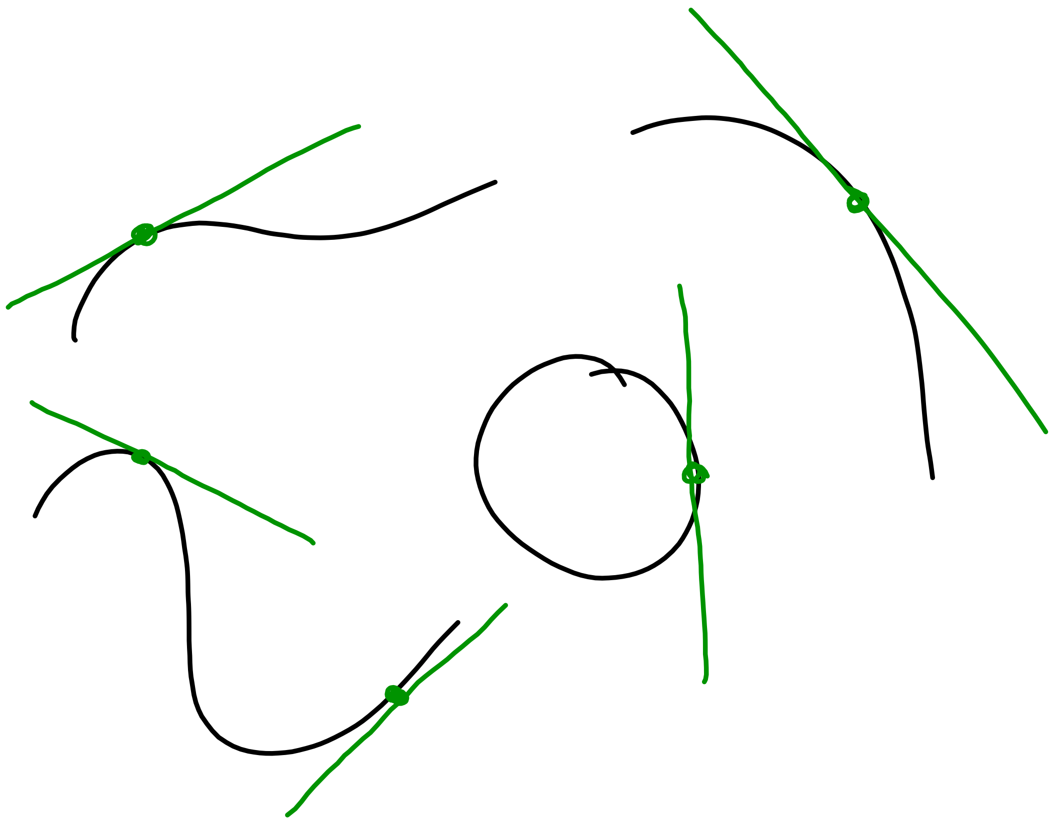
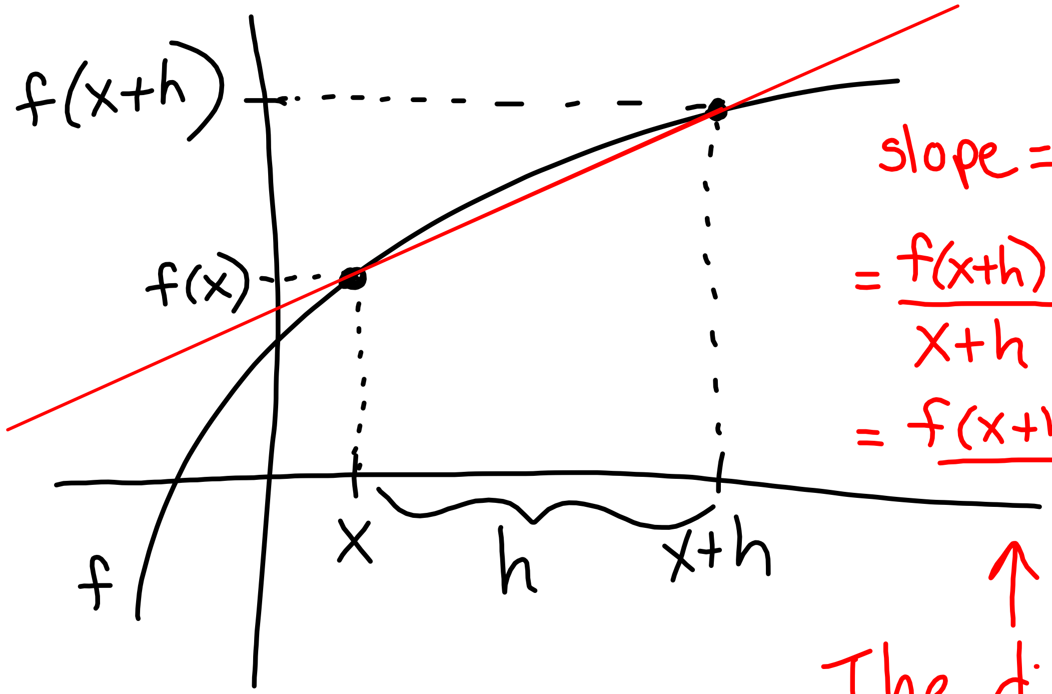


as x approaches...	$f(x)$ approaches...
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines



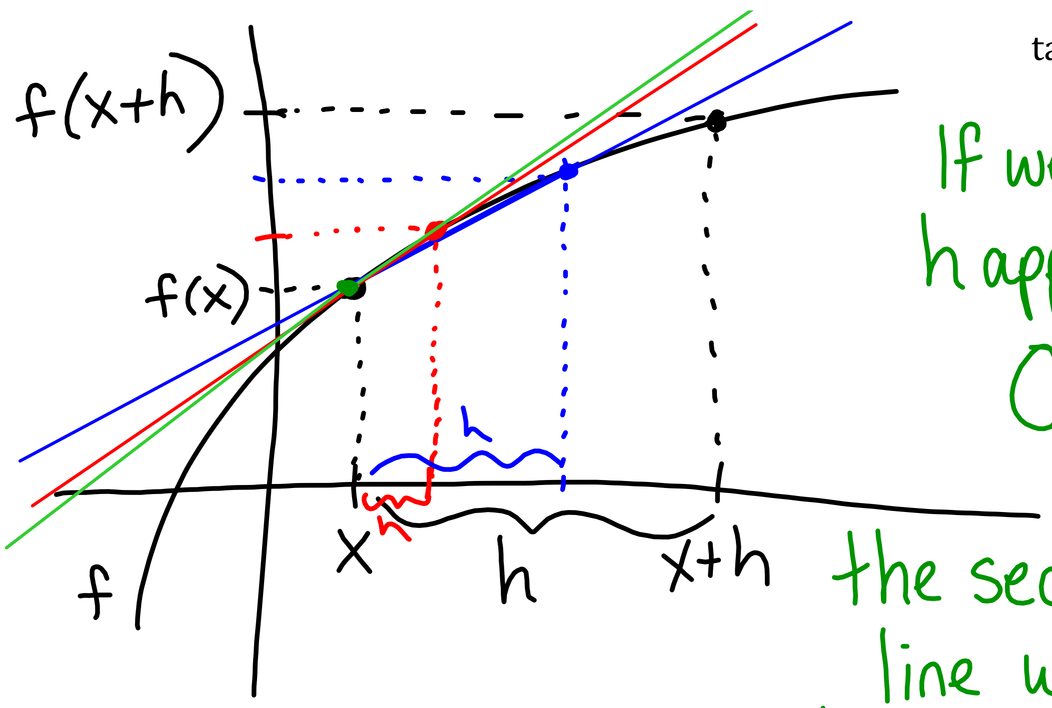
secant line



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

The difference quotient

tangent line



If we let h approach 0, the secant line will better & better approximate the tangent line

Δx "delta x"

change in x

$$\frac{f(x+\Delta x)-f(x)}{\Delta x}=\frac{f(x+\overset{\Delta x}{h})-f(x)}{\underset{\Delta x}{h}}$$

1.2

$$f(x)=\frac{x-2}{x^2-4}, \quad x\neq 2,-2$$

What happens to *f(x)* as *x* approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.2564	0.2506	0.2501	?	0.2499	0.2494	0.2439

$f(x)\to 0.25$ as $x\to 2$

Informal Description of the Limit

particular y-value

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

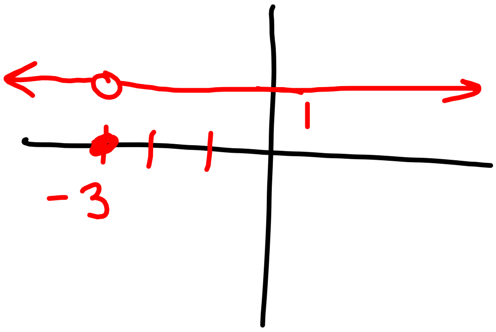
A function can be undeined for a certain value of *c* with the limit as *x* approaches *c* still deined.

$$\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3} = -0.25$$

$$f(x)=\begin{cases}1, & x\neq -3\\ 0, & x=-3\end{cases}$$

$y=1$

$f(-3)=0$

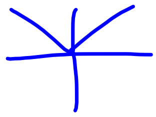


$$\lim_{x\to -3} f(x)=\boxed{1}$$

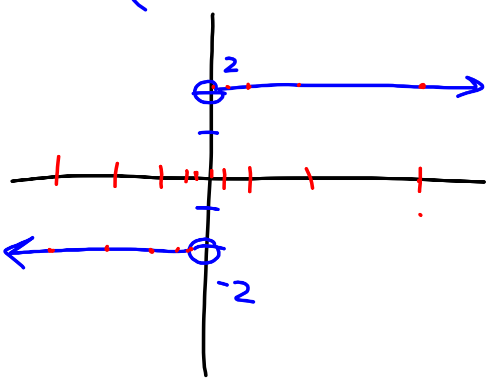
The limit is not necessarily equal to the function value.

$$\lim_{x\to 0}\frac{|2x|}{x}$$

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$$



$$\frac{|2x|}{x}=\begin{cases}\frac{2x}{x}=2, & x>0\\ \frac{-2x}{x}=-2, & x<0\end{cases}$$



$\lim_{x\to 0} f(x)=?$

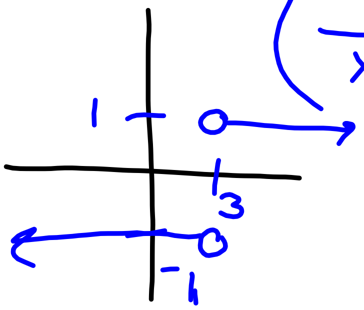
does not exist

$\lim_{x\to 0^+} f(x)=2$

$\lim_{x\to 0^-} f(x)=-2$

$$\lim_{x\to 3} \frac{|x-3|}{x-3} = \text{does not exist!}$$

$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3}, & x-3>0 \\ \frac{-(x-3)}{x-3}, & x-3<0 \end{cases} = \begin{cases} 1, & x>3 \\ -1, & x<3 \end{cases}$$

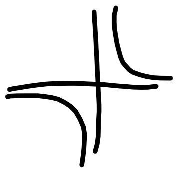


$$\lim_{x\to 3^+} f(x) = 1$$

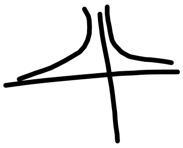
$$\lim_{x\to 3^-} f(x) = -1$$

$$\lim_{x\to 0} \frac{1}{x^4}$$ does not exist

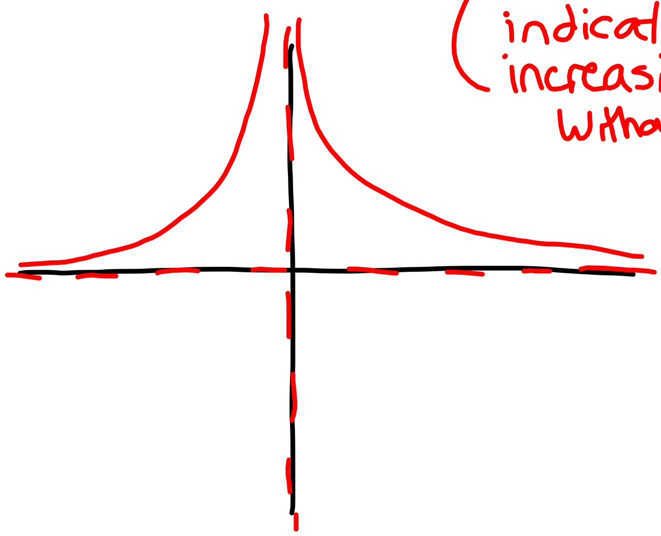
(= ∞ to indicate f is increasing without bound)



$\frac{1}{x^{odd}}$



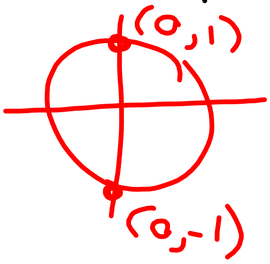
$\frac{1}{x^{even}}$

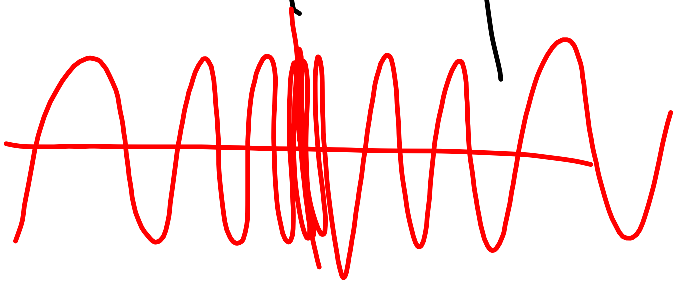


f is not approaching any real # y-value

$$\lim_{x \to 0} \sin\frac{1}{x}$$ does not exist (function oscillates

x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin\frac{1}{x}$	1	-1	1	-1	1	-1





"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

Limit does not exist anywhere!

Homework:
1.2 #1-7odd,9-18all