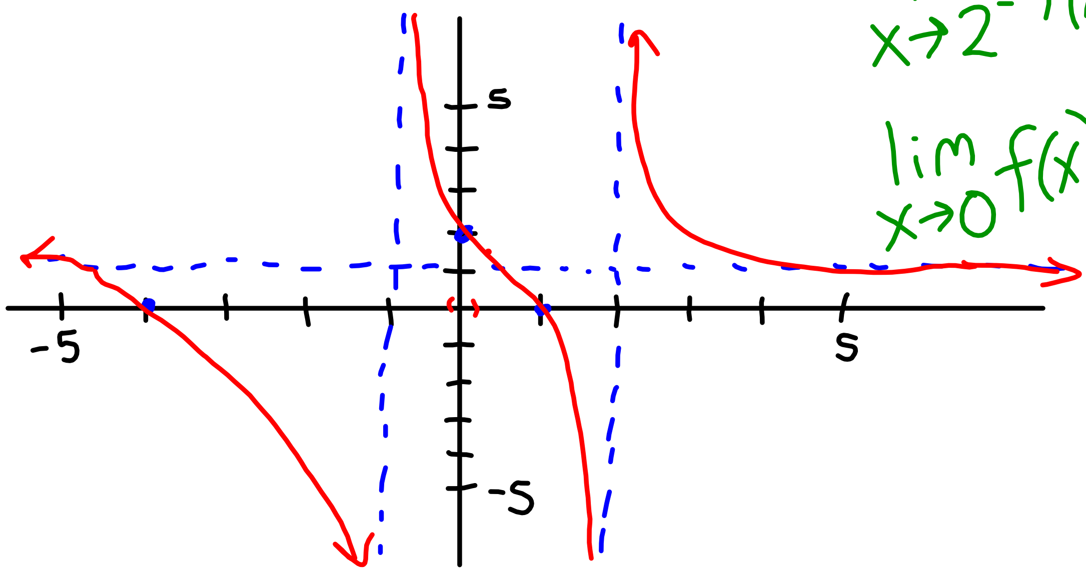
Graph the rational function.

$$f(x) = \frac{(x+4)(x-1)}{(x-2)(x+1)} \approx \frac{x^2}{x^2} = 1$$

$\lim\limits_{x \to \infty} f(x) = \boxed{1}$

$\lim\limits_{x \to 2^-} f(x) = \boxed{-\infty}$

$\lim\limits_{x \to 0} f(x) = \boxed{2}$

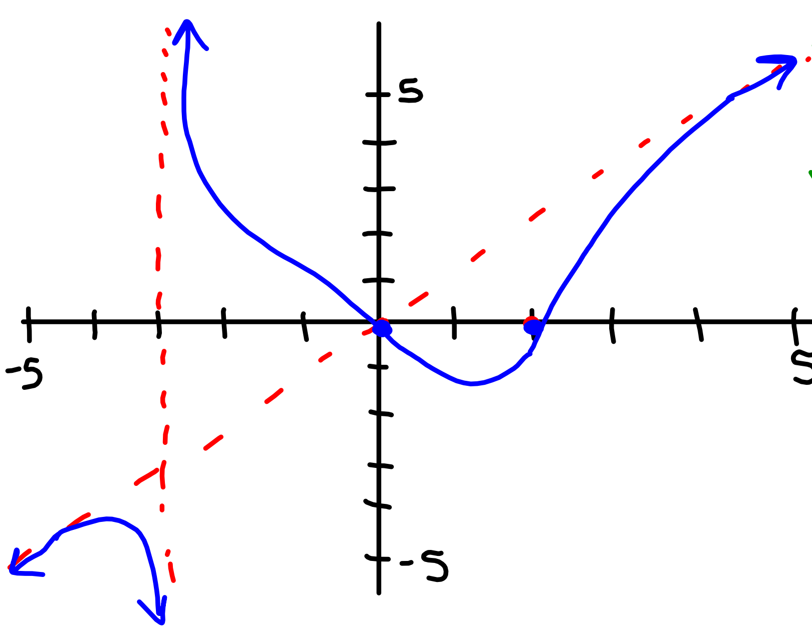


$$f(x) = \frac{x(x-2)}{x+3} \approx \frac{x^2}{x} = x$$

$\lim\limits_{x \to -\infty} f(x) = \boxed{-\infty}$

$\lim\limits_{x \to -3^+} f(x) = \boxed{+\infty}$

$\lim\limits_{x \to 2} f(x) = \boxed{0}$



Building up to the $\epsilon - \delta$ Definition of the Limit

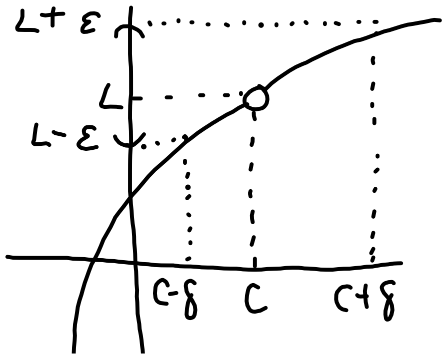
ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x \to c} f(x) = L$ particular y-value

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.

particular x-value



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L - \varepsilon, L + \varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c - \delta, c)$ or $(c, c + \delta)$.

$0 < |x - c| < \delta$

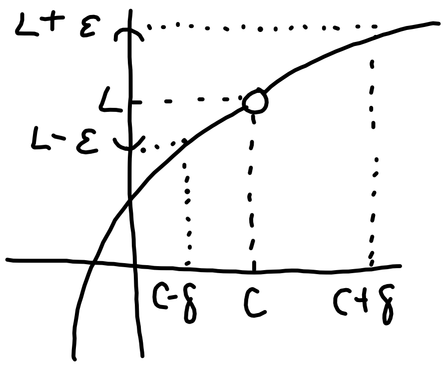
The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

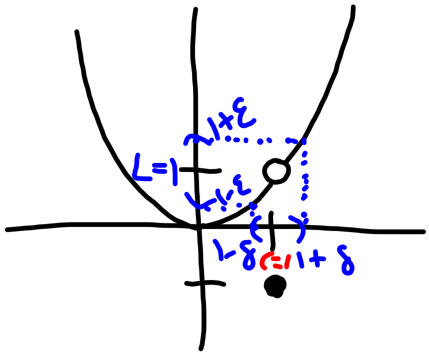
means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.



$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.



$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = \boxed{1}$$

c

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = 2x - 1$ $\quad \lim_{x \to 4} f(x) = 2(4) - 1 = 7$

Find $\lim_{x \to 4} f(x)$ and prove that is the limit using the ε-δ definition.

$L = 7$; $c = 4$; $f(x) = 2x - 1$

Let $\varepsilon > 0$ be given.

$|f(x) - L| = |2x - 1 - 7| = |2x - 8| = |2(x-4)|$

$= 2|x - 4|$ $\quad$ We want $2|x-4| < \varepsilon$

$|x-4| < \varepsilon/2$

Take $\delta = \varepsilon/2$.

Then whenever $0 < |x - 4| < \delta$, we have

$|2x - 1 - 7| = 2|x - 4| < 2 \cdot \delta = 2 \cdot \varepsilon/2 = \varepsilon$, i.e.

$|f(x) - L| < \varepsilon$.

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x+3$; find $\lim_{x \to 1} f(x)$ & find a δ.

$\lim_{x \to 1} f(x) = -5(1)+3 = -2$

$L = -2$; $C = 1$

$|f(x) - L| = |-5x+3-(-2)| = |-5x+5| =$

$= |-5(x-1)| = 5|x-1| < \varepsilon$

$|x-1| < \boxed{\varepsilon/5 = \delta}$

Prove that the limit is L using the $\varepsilon - \delta$ definition of the limit.

$L = -1$

$C = -3$

28. $\lim_{x \to -3} (2x + 5) = 2(-3)+5 = -1$

$|f(x) - L| = |2x+5-(-1)| = |2x+6| =$

$= 2|x+3| = 2|x-(-3)| < \varepsilon$

$|x-(-3)| < \boxed{\varepsilon/2 = \delta}$

Proof:

Given $\varepsilon > 0$. Take $\delta = \varepsilon/2$. Then

whenever $|x-(-3)| < \delta$, we have

$|2x+5-(-1)| = 2|x-(-3)| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$

i.e. $|f(x) - L| < \varepsilon$.

Find δ for $\varepsilon = 0.01$

24. $\lim_{x \to 4} \left(4 - \frac{x}{2}\right)$

Find δ for $\varepsilon = 0.01$

26. $\lim_{x \to 5} (x^2 + 4)$

Homework:

Already assigned:
1.2 #1-7odd,9-18all

New:
1.2 #23, 25, 27, 29, 30, 31
and watch all of the Khan Academy epsilon-delta videos!