

Find δ for $\varepsilon = 0.01$

$$24. \lim_{x \rightarrow 4} \left(4 - \frac{x}{2}\right) = 4 - \frac{4}{2} = 4 - 2 = 2$$

$$\varepsilon = 0.01 ; f(x) = 4 - \frac{x}{2} ; L = 2 ; c = 4$$

$$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| \rightsquigarrow \text{want } \begin{matrix} k |x-c| \\ k |x-4| \end{matrix}$$

$$\left|2 - \frac{x}{2}\right| = \left|-\frac{1}{2}(-4+x)\right| = \frac{1}{2}|x-4| < 0.01$$

Given $\varepsilon > 0$; $\exists \delta > 0$ s.t.
 $|x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon. \Rightarrow |x-4| < \boxed{0.02}$
 δ''

Find δ for $\varepsilon = 0.01$ $f(x) = x^2 + 4$; $c = 5$

$$26. \lim_{x \rightarrow 5} (x^2 + 4) = 5^2 + 4 = 29 = L$$

$$|f(x) - L| = |x^2 + 4 - 29| = |x^2 - 25| =$$

$$= |(x+5)(x-5)| = (x+5)|x-5|$$

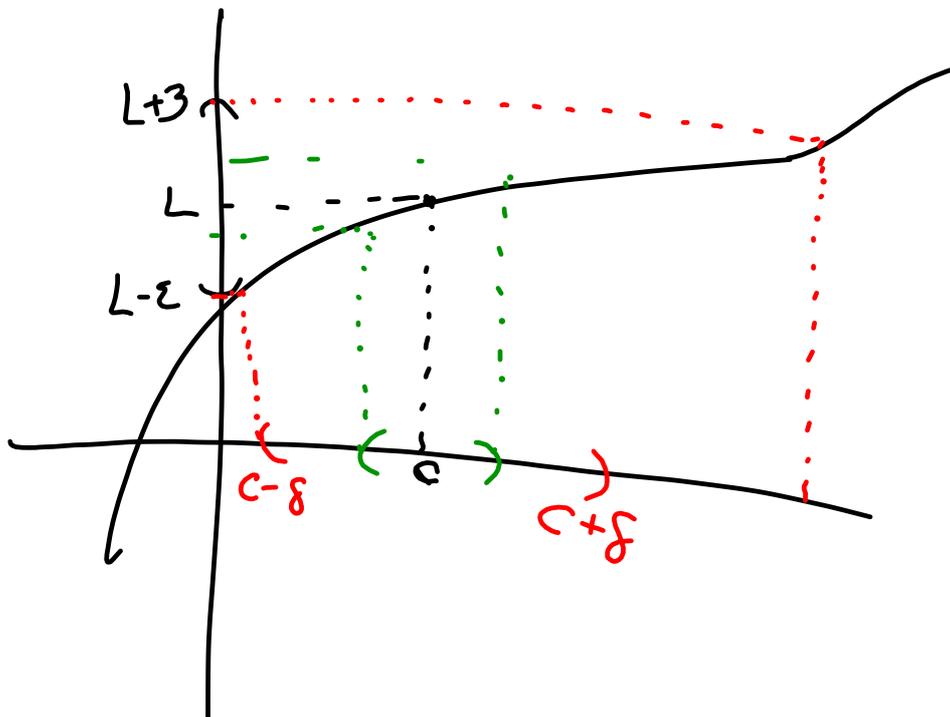
$$= (x+5)|x-5| < 13|x-5| < 0.01$$

↑
since $x > 0$
 $x+5 > 0$

$$\Rightarrow |x-5| < \boxed{\frac{0.01}{13}}$$

↑
since x is really close to 5
 $x+5 < 13$

$$\delta = \boxed{\frac{0.01}{13}}$$



$$\lim_{x \rightarrow c} 27$$

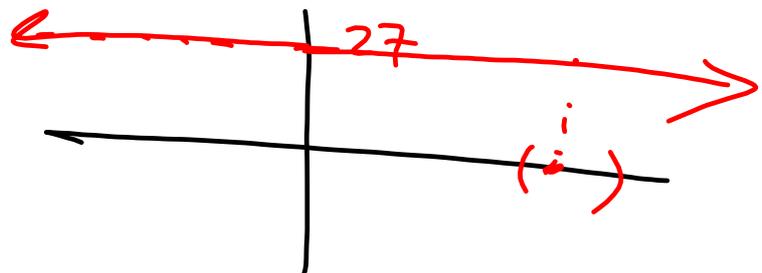
Let $\epsilon > 0$.

$$L = 27$$

$$|f(x) - L| = 0 < \epsilon$$

for any $\delta > 0$

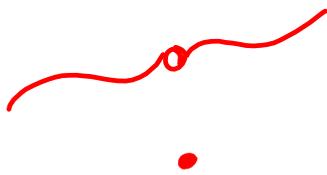
$$y = 27$$



1.3 Evaluating Limits Analytically

$$\text{If } \lim_{x \rightarrow c} f(x) = f(c),$$

we say that $f(x)$ is continuous at c .



$$\leftarrow \lim_{x \rightarrow c} f(x) \neq f(c)$$

Basic Limits

$$a, c \in \mathbb{R}$$

$$n \in \mathbb{N}$$

$$\lim_{x \rightarrow c} a = a$$

$$\lim_{x \rightarrow c} x = c$$

$$\lim_{x \rightarrow c} x^n = c^n$$

$$\lim_{x \rightarrow 5} (-3) = \boxed{-3}$$

$$\lim_{x \rightarrow -\pi} x = \boxed{-\pi}$$

$$\lim_{x \rightarrow -1} x^5 = \boxed{-1}$$

Theorem 1.2 more properties of Limits
 $b, c \in \mathbb{R}$, $n > 0$ an integer, f & g - functions
 $\lim_{x \rightarrow c} f(x) = L$; $\lim_{x \rightarrow c} g(x) = K$

1. scalar multiple

$$\lim_{x \rightarrow c} [bf(x)] = bL$$

2. sum or difference

$$\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$$

3. product

$$\lim_{x \rightarrow c} [f(x)g(x)] = LK$$

4. quotient

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{K}, K \neq 0$$

5. power

$$\lim_{x \rightarrow c} [f(x)]^n = L^n \quad (\text{follows from \#3})$$

polynomials, rational functions,
 $\sqrt[n]{x}$, $f(g(x))$, sin, cos, etc.

1.3

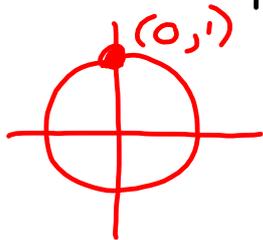
$$12. \lim_{x \rightarrow 1} (3x^3 - 2x^2 + 4)$$

$$= 3(1)^3 - 2(1)^2 + 4 = \boxed{5}$$

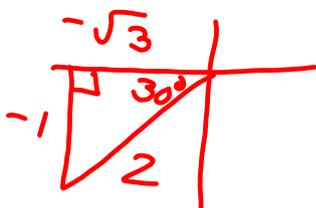
$$18. \lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4}$$

$$= \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \boxed{-2}$$

$$30. \lim_{x \rightarrow 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = \boxed{1}$$



$$36. \lim_{x \rightarrow 7} \sec \left(\frac{\pi x}{6} \right) = \sec \frac{7\pi}{6} = \boxed{-\frac{2}{\sqrt{3}}}$$



$$38. \lim_{x \rightarrow c} f(x) = \frac{3}{2} \quad ; \quad \lim_{x \rightarrow c} g(x) = \frac{1}{2}$$

$$(a) \lim_{x \rightarrow c} [4f(x)] = 4 \cdot \lim_{x \rightarrow c} f(x) = 4 \cdot \frac{3}{2} = \boxed{6}$$

$$(b) \lim_{x \rightarrow c} [f(x) + g(x)] = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = \boxed{2}$$

$$(c) \lim_{x \rightarrow c} [f(x)g(x)] = \frac{3}{2} \cdot \frac{1}{2} = \boxed{\frac{3}{4}}$$

$$(d) \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2} \cdot \frac{2}{1} = \boxed{3}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+1)}{x-3} =$$

$$\frac{(x-3)(x+1)}{x-3} = x+1, \quad x \neq 3$$

$$\rightarrow = \lim_{x \rightarrow 3} (x+1) = 3+1 = \boxed{4}$$

$$\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x \rightarrow 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}} = \sqrt{4} + 2 = \boxed{4}$$

$$\begin{aligned}
 &\text{Given } f(x) = 2x^2 + 3x + 1 \\
 &\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2 + 3x + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + 3h - \cancel{2x^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 + 3h}{h} = \lim_{h \rightarrow 0} \cancel{h} \frac{(4x + 2h + 3)}{\cancel{h}} \\
 &= \lim_{h \rightarrow 0} (4x + 2h + 3) = \boxed{4x + 3}
 \end{aligned}$$

$$\begin{aligned}
 &f(x) = x^3 \qquad (a+b)^3 \\
 &\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \qquad a^3 + 3a^2b + 3ab^2 + b^3 \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + \cancel{h^3}}{h} \qquad \begin{array}{c} | \quad | \quad | \quad | \\ | \quad 1 \quad 2 \quad | \quad (a+b)^1 \\ | \quad 3 \quad 3 \quad | \quad (a+b)^2 \\ | \quad 4 \quad 6 \quad 4 \quad | \quad (a+b)^3 \end{array} \\
 &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{\cancel{h}} = \boxed{3x^2}
 \end{aligned}$$

Homework:

Already assigned:

1.2 #1-7odd,9-18all

1.2 #23, 25, 27, 29, 30, 31 epsilon-delta

New:

1.3 #11,17,27-35odd, 39 - 61 odd