

Homework questions?

1.3

77. $\lim_{t \rightarrow 0} \frac{\sin 3t}{2t}$

$$\boxed{\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1}$$

$$= \lim_{t \rightarrow 0} \frac{\sin 3t}{3t} \cdot \frac{3}{2}$$

$$= \left(\lim_{t \rightarrow 0} \frac{\sin 3t}{3t} \right) \cdot \left(\lim_{t \rightarrow 0} \frac{3}{2} \right)$$

$$= 1 \cdot \frac{3}{2}$$

$$= \boxed{\frac{3}{2}}$$

$$\begin{aligned} \sin 3t &\cancel{\times} (\sin t)^3 \\ \sin(2t+t) &= \\ \underline{\sin 2t \cos t +} & \\ \underline{\cos 2t \sin t} &= \end{aligned}$$

1.4

59. $f(x) = \begin{cases} 2, & x \leq -1 \\ ax+b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

$$2 = a(-1) + b$$

$$-2 = a(3) + b$$

$$\left. \right\}$$

solve linear
system of equations
to find a & b .

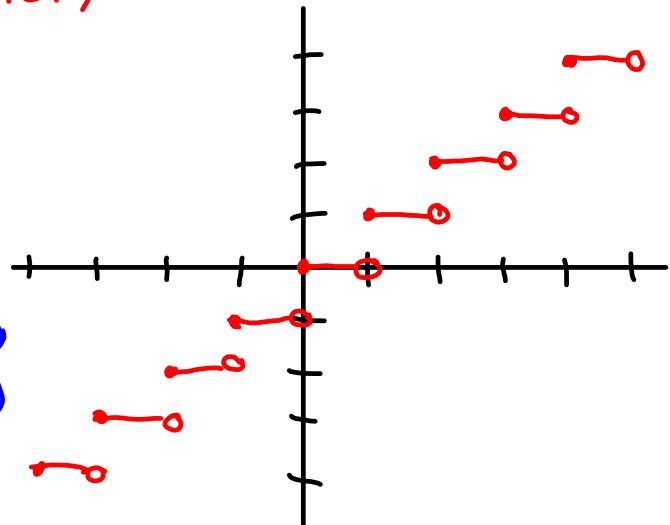
The Greatest Integer Function

$\lfloor x \rfloor$ = the greatest integer less than or equal to x
 "step function"

$\lfloor x \rfloor$

$$\lfloor \pi \rfloor = 3$$

$$\lfloor -7.12 \rfloor = -8$$



$$22. \lim_{x \rightarrow 2^+} 2x - \lfloor x \rfloor$$

$$= \lim_{x \rightarrow 2^+} (2x) - \lim_{x \rightarrow 2^+} \lfloor x \rfloor$$

$$= 4 - 2$$

$$= \boxed{2}$$

$$24. \lim_{x \rightarrow 1} \left(1 - \left\lfloor -\frac{x}{2} \right\rfloor \right)$$

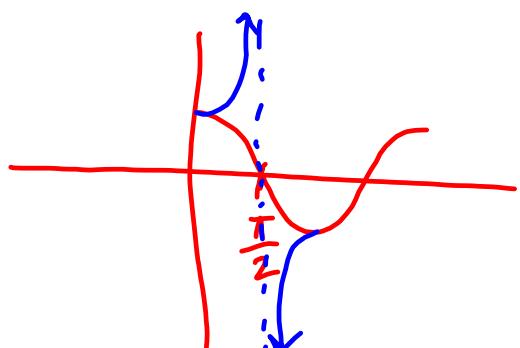
$$= \lim_{x \rightarrow 1} (1) - \lim_{x \rightarrow 1} \left\lfloor -\frac{x}{2} \right\rfloor$$

$$= 1 - (-1) = \boxed{2}$$

$$20. \lim_{x \rightarrow \frac{\pi}{2}} \sec x$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{\cos x}$$

= does not exist



$$52. f(x) = \tan \frac{\pi x}{2}$$

discuss the (dis)continuity

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$\tan \theta$ is undefined when $\cos \theta = 0$

i.e. when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \text{ etc.}$
(odd multiples of $\frac{\pi}{2}$)

$f(x) = \tan \frac{\pi x}{2}$ has vertical asymptotes (nonremovable discontinuities) at all odd integers x .

f is continuous on all intervals of the form $(2k+1, 2k+3)$, $k \in \mathbb{Z}$

$$62. f(x) = \frac{1}{\sqrt{x}}, g(x) = x-1$$

Discuss the continuity of $f(g(x))$.

$$f(g(x)) = \frac{1}{\sqrt{x-1}}$$

domain:
 $x-1 > 0$
 $x > 1$

$f(g(x))$ is continuous on its domain $(1, \infty)$

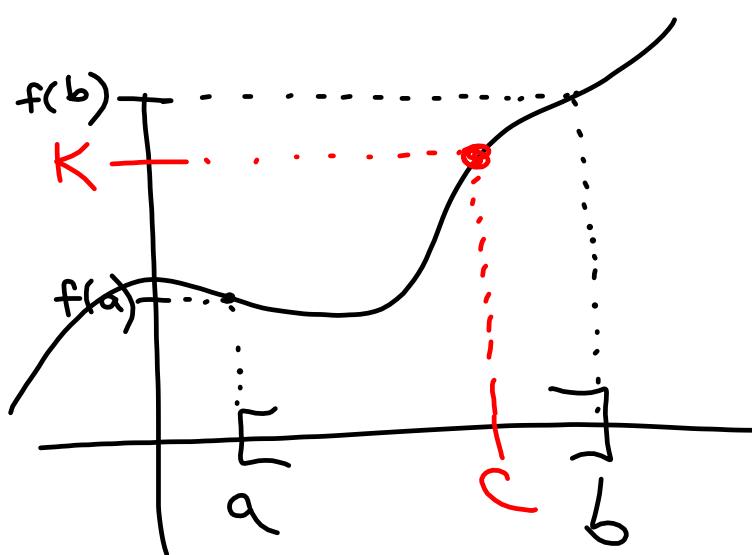
$$6A. \ f(x) = \sin x ; \ g(x) = x^2$$

discuss the continuity of $f(g(x))$

$$\left. \begin{array}{l} f(g(x)) = \sin(x^2) \\ g(f(x)) = (\sin x)^2 \end{array} \right\} \text{continuous on } (-\infty, \infty)$$

Intermediate Value Theorem

If f is continuous on the closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$.



Does the IVT guarantee a zero in the given interval?

$$76. f(x) = x^3 + 3x - 2 \quad , [0, 1]$$

f is continuous on $[0, 1]$

$$f(0) = 0^3 + 3(0) - 2 = -2 < 0$$

$$f(1) = 1^3 + 3(1) - 2 = 2 > 0$$

Let's find it!

$$x^3 + 3x - 2 = 0$$

;) Never mind...
(use a TI-89!)

$$84. f(x) = x^2 - 6x + 8 ; [0, 3] \cdot f(c) = 0$$

f is continuous on $[0, 3]$

$$f(0) = 8 > 0$$

$$f(3) = -1 < 0$$

set $f(x) = 0$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$\text{X} = 2, \text{X} = 4$$

not in $[0, 3]$

Yes, the IVT guarantees a $c \in [0, 1]$ st. $f(c) = 0$.

$$86. f(x) = \frac{x^2+x}{x-1} , \left[\frac{5}{2}, 4 \right] , f(c) = 6$$

f is continuous on $\left[\frac{5}{2}, 4 \right]$

$$f\left(\frac{5}{2}\right) = \frac{\left(\frac{5}{2}\right)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \frac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \frac{35}{4} \cdot \frac{2}{3} = \frac{35}{6} < 6$$

$$f(4) = \frac{4^2+4}{4-1} = \frac{20}{3} > 6 \Rightarrow \text{IVT guarantees } a < c.$$

$$\frac{x^2+x}{x-1} = 6$$

$$x^2 + x = 6x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$\cancel{x=2} \quad x=3$$

Homework for Test #1:

HW#1 (submitted Mon. 11/12)

- 1.2 #1-7odd, 9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta
- 1.3 #11, 17, 27-35odd,

HW#2 (due Mon. 11/18?)

- 1.3 #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- **1.4 #19, 21, 23, 51, 63, 69, 71, 83, 85**
- **Test #1 Practice Problems**

Quiz #1 - Take-home; due Friday 11/15

Test #1 - Wednesday, 11/20