Turn in Take-home Quiz #1!

1. $\lim_{x\to -3} f(x)$, where $f(x) = \begin{cases} 4-x, x \neq -3 \\ 0, \quad x = -3 \end{cases}$

$= \boxed{7}$

2. $\lim_{x\to 3} \frac{|x-3|}{3-x}$

$\frac{|x-3|}{3-x} = \begin{cases} \frac{x-3}{3-x} = -1, & x>3 \\ \frac{-(x-3)}{3-x} = 1, & x<3 \end{cases}$

$\boxed{\text{does not exist}}$

3. $\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

$= \boxed{-4}$

4. $\lim_{x\to 2} \frac{3x}{\sqrt{x+7}} = \frac{3.2}{\sqrt{2+7}} = \boxed{2}$

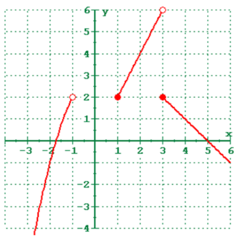
5. $\lim_{x\to 0} \frac{\sin x}{x} = \boxed{1}$

6. $\lim_{x\to 0} \frac{1-\cos x}{x} = \boxed{0}$

7. $\lim_{x\to -1} \frac{\sqrt{x+2}-1}{x+1} \cdot \frac{\sqrt{x+2}+1}{\sqrt{x+2}+1}$

$= \lim_{x\to -1} \frac{\cancel{x+2-1}\ 1}{\cancel{(x+1)}(\sqrt{x+2}+1)} = \boxed{\frac{1}{2}}$

8. Fill in the blank: A function f is continuous at c if $\lim_{x\to c} f(x) =$ $f(c)$



Assume that the graph to the left is $f(x)$.

Find:

9. $\lim_{x\to 3^+} f(x)$ 2

10. $\lim_{x\to -1^-} f(x)$ 2

Discuss the continuity of the following functions. State which discontinuities are removable and which are non-removable, and state the intervals on which the function is continuous.

11. $f(x) = \frac{x-5}{x^2-3x-10} = \frac{x-5}{(x-5)(x+2)}$

removable disconitnuity @ $x=5$

non-removable discontinuity @ $x=-2$

f is continuous on $(-\infty,-2)\cup(-2,5)\cup(5,\infty)$

12. $f(x) = \begin{cases} \frac{1}{2}x+1, & x \le 4 \\ 3-x, & x > 4 \end{cases}$

$\frac{1}{2}(4)+1=3$

$3-4=-1$

non-removable (jump) discontinuity @ $x=4$

f is continuous on $(-\infty,4] \cup (4,\infty)$

1.4

85. $f(x)=x^3-x^2+x-2$; $[0,3]$; $f(c)=4$

f is cts on $[0,3]$ ✓
$f(0)=-2<4$
$f(3)=19>4$

IVT guarantees a c in $[0,3]$ s.t. $f(c)=4$.

$x^3-x^2+x-2=4$
$x^3-x^2+x-6=0$
$2^3-2^2+2-6=8-4+2-6=0$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & \boxed{0} \end{array}$$

$\boxed{c=2}$

$(x-2)\underbrace{(x^2+x+3)}_{\text{complex zeros}}=0$

1.5

Infinite Limits

$$\lim_{x \to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x)=\frac{1}{x(x+3)}$ has vertical asymptotes at $x=0$ and $x=3$

$f(x)=\frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x=0$ and a hole at $x=-3$

Rules involving infinite limits

Let $\lim_{x \to c} f(x) = \infty$ *and* $\lim_{x \to c} g(x) = L$

1. $\lim_{x \to c}[f(x) \pm g(x)] = \infty$

2. $\lim_{x \to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$

Find the vertical asymptotes (if any).

14. $f(x) = \frac{-4x}{x^2+4}$ none!

24. $h(x) = \frac{x^2-4}{x^3+2x^2+x+2}$ none!

$\frac{(x-2)\cancel{(x+2)}}{x^2(x+2)+1(x+2)} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x^2+1)}$

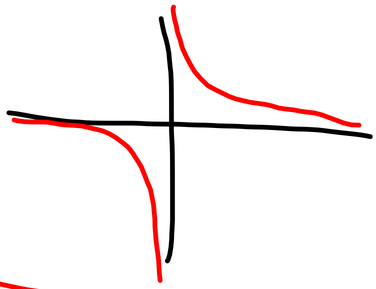
28. $g(\theta) = \frac{\tan\theta}{\theta} = \frac{\sin\theta}{\theta \cdot \cos\theta}$

$\theta = 0$; & all odd multiples of $\frac{\pi}{2}$

42. $\lim_{x\to 0^-}\left(x^2-\frac{1}{x}\right)$

$= \lim_{x\to 0^-} x^2 - \lim_{x\to 0^-}\frac{1}{x}$

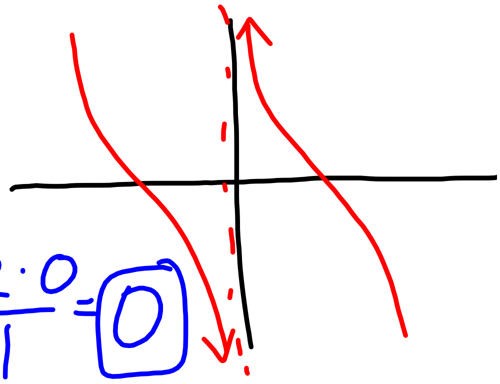
$= 0-(-\infty) = \boxed{\infty}$



46. $\lim_{x\to 0}\frac{x+2}{\cot x} \to \frac{2}{\infty}$

$= \boxed{0}$

$\lim_{x\to 0}\frac{x+2}{\left(\frac{\cos x}{\sin x}\right)} = \lim_{x\to 0}\frac{(x+2)(\sin x)}{\cos x} = \frac{2\cdot 0}{1} = \boxed{0}$

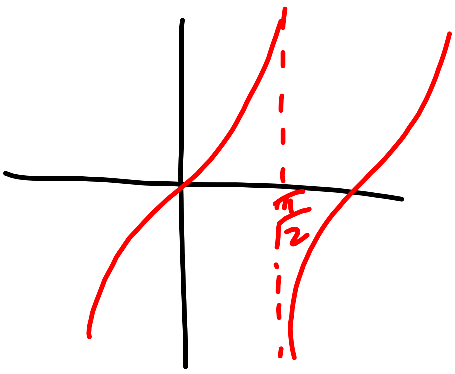


48. $\lim_{x\to \frac{1}{2}} x^2 \tan \pi x$

$= \left(\lim_{x\to\frac{1}{2}} x^2\right)\left(\lim_{x\to\frac{1}{2}} \tan\pi x\right)$

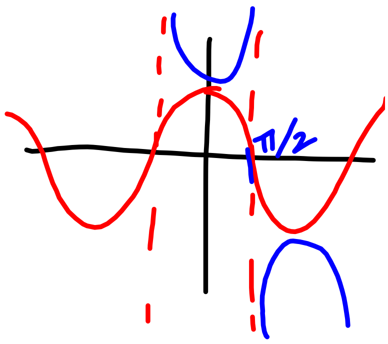
$= \frac{1}{4}\cdot\lim_{x\to\frac{\pi}{2}}\tan x$

does not exist



52. $\lim_{x\to 3^+}\sec\frac{\pi x}{6} = \lim_{x\to\frac{\pi}{2}^+}\sec x$

$= \boxed{-\infty}$



2.1 The Derivative & The Tangent Line Problem

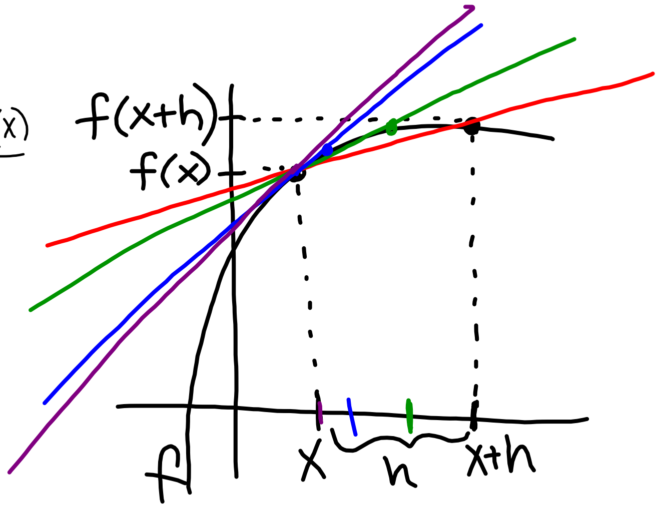
secant line
crosses through a function at two points

slope of the secant line:

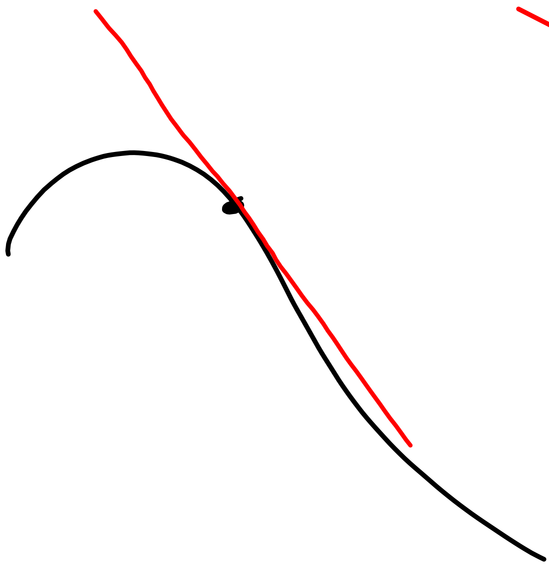
$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

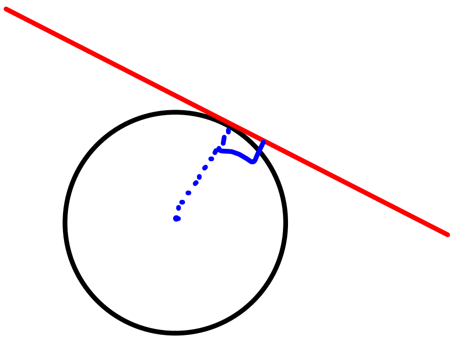
what happens as $h \to 0$?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.





$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\boldsymbol{f'(x)}$	"f prime of x"
$\dfrac{\boldsymbol{dy}}{\boldsymbol{dx}}$	"derivative of y with respect to x"
$\boldsymbol{y'}$	"y prime"
$\dfrac{\boldsymbol{d}}{\boldsymbol{dx}}[\boldsymbol{f(x)}]$	"the derivative with respect to x of $f(x)$"
$\boldsymbol{D_x[y]}$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

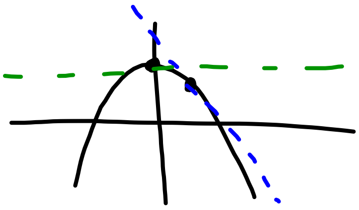
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

8. $g(x) = 5 - x^2$

find slope of tangent line at the points $(2,1)$ & $(0,5)$



$$m = f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= g'(c) = \lim_{h\to 0} \frac{g(c+h) - g(c)}{h}$$

$(2,1)$:

$$g'(2) = \lim_{h\to 0} \frac{5-(2+h)^2 - 1}{h} = \lim_{h\to 0} \frac{4-(4+2h+h^2)}{h}$$

$$= \lim_{h\to 0} \frac{-2h-h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-2-h)}{\cancel{h}} = \boxed{-2}$$

$(0,5)$:

$$g'(0) = \lim_{h\to 0} \frac{5-h^2-5}{h} = \lim_{h\to 0} \frac{-h^2}{h} = \lim_{h\to 0} -h = \boxed{0}$$

Homework for Test #1 (Wednesday, 11/20):

HW#1 (submitted Mon. 11/12)

- 1.2 #1-7odd,9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta
- 1.3 #11,17,27-35odd,

HW#2 (due Mon. 11/18)

- 1.3 #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- 1.4 #19,21,23,51,63,69,71,83,85
- Test #1 Practice Problems (handout; not listed on your syllabus!)

HW #3 (due Wed. 11/20, test day)

- 1.5 (ininite limits) - p.85 #1-51odd
- Ch 1 review pp. 88-89
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)

HW #4 (not due until after the test, but will still help you with limits that will be on the test)

- 2.1 (derivative deinition) - p.101-102 #1-23odd