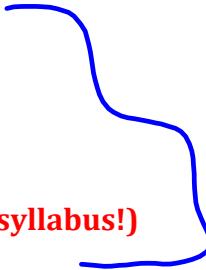


Homework for Test #1 (Wednesday, 11/20):**HW#1 (submitted Mon. 11/12)**

- 1.2 #1-7odd, 9-18all
- 1.2 #23, 25, 27, 29, 30, 31 epsilon-delta
- 1.3 #11, 17, 27-35odd,

HW#2 (due Mon. 11/18)

- 1.3 #39-61odd (<-- not listed on your syllabus!)
- 1.3 #67-77odd; 87, 88 (<-- not listed on your syllabus!)
- 1.4 #7-17odd; 25-28all; 39-47odd; 57, 59
- 1.4 #19, 21, 23, 51, 63, 69, 71, 83, 85
- **Test #1 Practice Problems (handout; not listed on your syllabus!)**

**HW #3 (due Wed. 11/20, test day)**

- 1.5 (infinite limits) - p.85 #1-51odd
- Ch 1 review pp. 88-89
- (*recommended* - Old Test #1 on web; *solutions can be found in course notes from last term*)

HW #4 (not due until after the test, but will still help you with limits that will be on the test)

- 2.1 (derivative definition) - p.101-102 #1-23odd

1. Find the domain and range of the function.

a. $y = x^2 + 1$

b. $y = -\sqrt{x-2}$

c. $y = \frac{1}{x+2}$

2. Determine the domain and range of the piecewise function, and evaluate the function as indicated.

$$f(x) = \begin{cases} 3x + 3 & x < 1 \\ -x^2 - 3, & x \geq 1 \end{cases}$$

a. domain

b. range

c. $f(-2)$ d. $f(3)$ e. $f(t^2 + 5)$ a. domain: $(-\infty, \infty)$

b. range

$3x + 3$

$(-\infty, \infty)$

$x < 1$

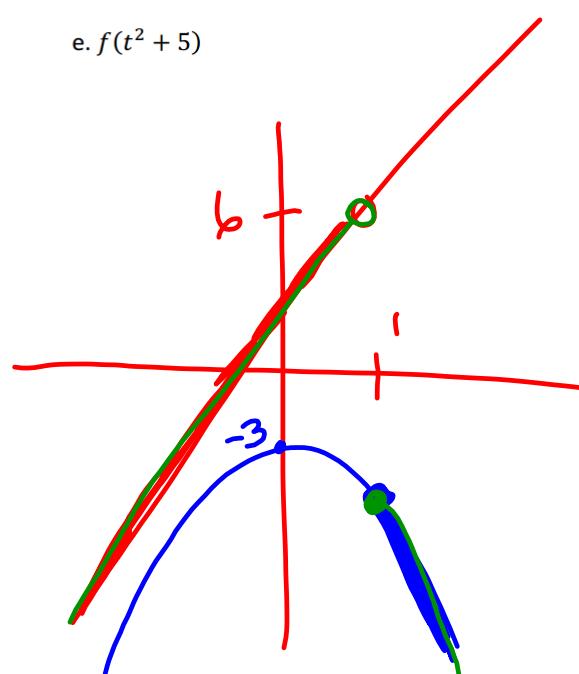
$(-\infty, 6)$

$-x^2 - 3$

$(-\infty, -3]$

$x \geq 1$

$(-\infty, -4]$



3. Find the composition functions $(f \circ g)$ and $(g \circ f)$ and state the domain of each composition function.

$$f(x) = x^2 + 1, \quad g(x) = \sin x$$

4. Find the composition functions $(f \circ g)$ and $(g \circ f)$ and state the domain of each composition function.

$$f(x) = x^2 + 3, \quad g(x) = \frac{1}{x-2}$$

$$(f \circ g)(x) = f(g(x)) = \left(\frac{1}{x-2}\right)^2 + 3$$

domain: $(-\infty, 2) \cup (2, \infty)$

$$(g \circ f)(x) = g(f(x)) = \frac{1}{x^2+3-2} = \frac{1}{x^2+1}$$

domain: $(-\infty, \infty)$

5. Find the limit L , then use the $\varepsilon - \delta$ definition to prove that the limit is L .

$$\lim_{x \rightarrow 2} (2x - 3) = L = 1; C = 2; f(x) = 2x - 3$$

Given $\varepsilon > 0$, we want to find a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - C| < \delta$.

$$|f(x) - L| = |2x - 3 - 1| = |2x - 4| = |2(x - 2)| = 2|x - 2|$$

We want $2|x - 2| < \varepsilon$, so take $\boxed{\delta = \varepsilon/2}$.

Then whenever $|x - 2| < \delta$, we have

$$|f(x) - L| = \dots = 2|x - 2| < 2 \cdot \delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon, \text{ i.e.}$$

$|f(x) - L| < \varepsilon$. Hence, $\lim_{x \rightarrow 2} (2x - 3) = 1$.

6. Find the limit.

a. $\lim_{x \rightarrow 2} (2x^3 - x + 5)$

$$2(2)^3 - 2 + 5$$

$$= \boxed{19}$$

b. $\lim_{x \rightarrow 2} \frac{x+4}{x^2+1}$

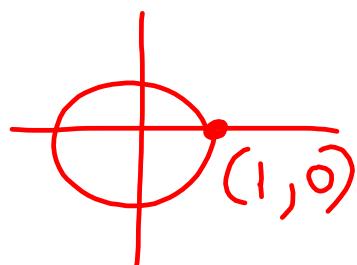
$$\frac{2+4}{2^2+1}$$

$$= \boxed{\frac{6}{5}}$$

c. $\lim_{x \rightarrow -2} \cos \pi x$

$$\cos(-2\pi)$$

$$= \boxed{1}$$



7. Use the given information to evaluate the given limits.

$$\lim_{x \rightarrow c} f(x) = -3, \quad \lim_{x \rightarrow c} g(x) = 5$$

a. $\lim_{x \rightarrow c} [2f(x) + \sqrt{g(x)}]$

$$= 2(-3) + \sqrt{5}$$

$$= \boxed{-6 + \sqrt{5}}$$

b. $\lim_{x \rightarrow c} [3f(x)\sqrt{g(x)}]$

$$= 3(-3)\sqrt{5}$$

$$= \boxed{-9\sqrt{5}}$$

8. Find the limit (if it exists).

$$\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^2 - 1} = \lim_{x \rightarrow -1} \frac{(x-2)(x+1)}{(x-1)(x+1)} = \frac{-1-2}{-1-1} = \frac{-3}{-2} = \boxed{\frac{3}{2}}$$

9. Find the limit (if it exists).

$$\lim_{x \rightarrow -1} \frac{x^2 - 9}{x^2 - 5x + 6} = \lim_{x \rightarrow -1} \frac{(x-3)(x+3)}{(x-3)(x-2)} = \frac{-1+3}{-1-2} = \boxed{\frac{2}{-3}}$$

$$= \frac{(-1)^2 - 9}{(-1)^2 - 5(-1) + 6}$$

$$= \frac{1-9}{1+5+6} = \frac{-8}{12} = \boxed{\frac{-2}{3}}$$

10. Find the limit (if it exists).

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x^2 - 1} \cdot \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2} = \lim_{x \rightarrow 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} =$$

$$= \lim_{x \rightarrow 1} \frac{1}{(x-1)(x+1)(\sqrt{x+3}+2)} = \frac{1}{(1-1)(1+1)(\sqrt{1+3}+2)} = \frac{1}{2(2+2)} = \boxed{\frac{1}{8}}$$

$$\boxed{\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1}$$

11. Determine the limit of the trigonometric function (if it exists).

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{2x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot \frac{5}{2} = 1 \cdot \frac{5}{2} = \boxed{\frac{5}{2}}$$

12. Determine the limit of the trigonometric function (if it exists).

$$\lim_{x \rightarrow 0} \frac{\tan^2 x}{2x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{2x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{2\cos^2 x}$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$1 \cdot 0 = \boxed{0}$$

$$\boxed{\lim_{x \rightarrow 0} \frac{1-\cos x}{x} = 0}$$

13. Determine the limit of the trigonometric function (if it exists).

$$\lim_{x \rightarrow 0} \frac{3(1 - \cos x)^2}{x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \cdot \frac{1 - \cos x}{1} \cdot 3$$

$$\downarrow \qquad \qquad \downarrow$$

$$0 \cdot 0 \cdot 3 = \boxed{0}$$

14. Find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}, \quad \text{where } f(x) = x + 1$$

$$\lim_{h \rightarrow 0} \frac{x+h+1 - (x+1)}{h} = \lim_{h \rightarrow 0} \frac{x+h+1-x-1}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = \boxed{1}$$

15. Find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}, \quad \text{where } f(x) = 2x^2 - 1$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 1 - (2x^2 - 1)}{h} = \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) - 1 - 2x^2 + 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - x^2 + 1}{h} = \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h} = \boxed{4x} \end{aligned}$$

16. Use the Squeeze Theorem to find $\lim_{x \rightarrow 0} f(x)$.

$$f(x) = x^2 \sin \frac{3}{x} \quad -1 \leq \sin \frac{3}{x} \leq 1$$

$$-x^2 \leq x^2 \sin \frac{3}{x} \leq x^2$$

$$\begin{array}{ccc} \lim_{x \rightarrow 0} (-x^2) & \leq & \lim_{x \rightarrow 0} (x^2 \sin \frac{3}{x}) & \leq \lim_{x \rightarrow 0} (x^2) \\ \parallel & & \boxed{0} \text{ (by squeeze thm)} & \parallel \end{array}$$

17. Use the Squeeze Theorem to find $\lim_{x \rightarrow 0} f(x)$.

$$2 - 3x^2 \leq f(x) \leq 2 + 5x^2$$

$$\begin{array}{ccc} \lim_{x \rightarrow 0} (2 - 3x^2) & \leq & \lim_{x \rightarrow 0} f(x) & \leq \lim_{x \rightarrow 0} (2 + 5x^2) \\ \parallel & & \boxed{2} \text{ (by squeeze thm)} & \parallel \end{array}$$

18. Find the limit (if it exists).

$$\lim_{x \rightarrow 4^+} \frac{|x-4|}{x-4} = \boxed{1}$$

$$\frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} = 1 & , x-4 > 0 \\ \frac{-(x-4)}{x-4} = -1 & , x-4 < 0 \end{cases}, \quad x > 4$$

19. Find the limit (if it exists).

$$\lim_{x \rightarrow 0^+} f(x), \quad f(x) = \begin{cases} 2x^2 + 2x + 1, & x \leq 0 \\ x - 3, & x > 0 \end{cases}$$

$$= 0 - 3 = \boxed{-3}$$

20. Find the limit (if it exists).

$$\lim_{x \rightarrow 2} f(x), \quad f(x) = \begin{cases} 10 - x, & x \leq 2 \\ x^2 + 2x, & x > 2 \end{cases}$$

$$10 - 2 = 8$$

$$2^2 + 2(2) = 8$$

$$\lim_{x \rightarrow 2} f(x) = \boxed{8}$$

8. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2, 3]$ such that $f(c) = -4$, and if so, find all such values of c .

$$f(x) = x^2 - 7x + 2$$

9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2}$$