The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

Alt form:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

$\frac{d}{dx}[x] = 1$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.2

22. $y = 5 + \sin x$

$y' = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \boxed{-\frac{15}{8}x^{-4} - 2\sin x}$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}} = 4x - \frac{1}{x^2} = \frac{4x^3 - 1}{x^2}$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = \boxed{36x - 45x^2}$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = \boxed{-\frac{2}{3}x^{-4/3} - 3\sin x}$

2.2 cont.

$s(t)$ = position

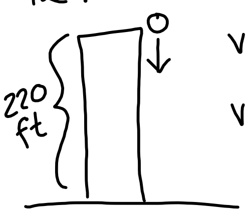
$v(t) = s'(t)$ = velocity

$a(t) = v'(t) = s''(t)$ = acceleration

average velocity: $\dfrac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92.



initial velocity $v_0 = -22$ ft/s

$v(3) = ?$

$v(t) = ?$ after falling 108 ft

$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$

$g = -9.8\ m/s^2$

$= -32\ ft/s^2$

v_0 = initial velocity

s_0 = inital position

g = acceleration due to gravity

$s(t) = -16t^2 - 22t + 220$

$v(t) = s'(t) = -32t - 22$

$v(3) = -32(3) - 22 = -96 - 22$

$= \boxed{-118 \text{ ft/s}}$

$-16t^2 - 22t + 220 = 108$

$0 = 16t^2 + 22t - 112$

$0 = 8t^2 + 11t - 56$

$\begin{array}{r} 56 \\ \times 32 \\ \hline 112 \\ 1680 \end{array}$

$t = \dfrac{-11 \pm \sqrt{121 - 4(8)(-56)}}{2(8)} = \dfrac{-11 \pm \sqrt{121 + 1792}}{16} = \dfrac{-11 \pm \sqrt{1913}}{16}$

$= \dfrac{-11 + \sqrt{1913}}{16} \approx 2.05\ s$

$v(2.05) = -32(2.05) - 22 = \boxed{-42.73 \text{ ft/s}}$

sphere volume: $V = \frac{4}{3}\pi r^3$

find the rate of change of volume w.r.t. radius when $r = 2$ cm.

$V = \frac{4}{3}\pi r^3 = V(r)$

$\frac{dV}{dr} = 4\pi r^2$ = surface area of a sphere!

$V'(2) = 4\pi(2)^2 = \boxed{16\pi \text{ cm}^2}$

2.3 Product & Quotient Rules

$[fg]' = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

$$\boxed{(fg)' = f'g + fg'}$$

$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

"low dee high less high dee low, draw the line and square below"

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1} \qquad \text{d/dx [c]=0}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'\big(g(x)\big)g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Thursday:
- product rule, quotient rule, chain rule

Friday:
- quiz on basic derivative rules (those stated above);
  - product, quotient, & chain rules - need to know statement (formula) only
  - all other rules should be applied to find derivatives

Find the derivative by the limit process:
2.1 #1-23odd

Find the equation of the tangent line:
2.1 #29-32

New!

Use the alternate form to find the derivative:
2.1 #61-69 odd

Describe the x-values where the function is differentiable (given a graph):
2.1 #71-79 odd

Find the derivative using the basic derivative rules we have learned so far:
2.2 #3-51 odd

Use the derivative to solve rate of change word problems:
2.2 #91-94; 101,102

New!

Work through intuitive exercises on **Khan Academy**:
- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives