

What happens if . . .

$$x^2y + y^2x = -2$$

how to find  $y'$ ?

## 2.5 Implicit Differentiation

$$\cancel{y = f(x)}$$

$y$  is a function of  $x$

$$\frac{d}{dx}[x] = 1 \quad ; \quad \frac{d}{dx}[y] = y'$$

$$6. \quad x^2y + y^2x = 2$$

$$\frac{d}{dx} [x^2y + y^2x] = \frac{d}{dx} [-2]$$

$$(x^2)' \cdot y + x^2 \cdot (y)' + (y^2)' \cdot x + y^2 \cdot (x)' = 0$$

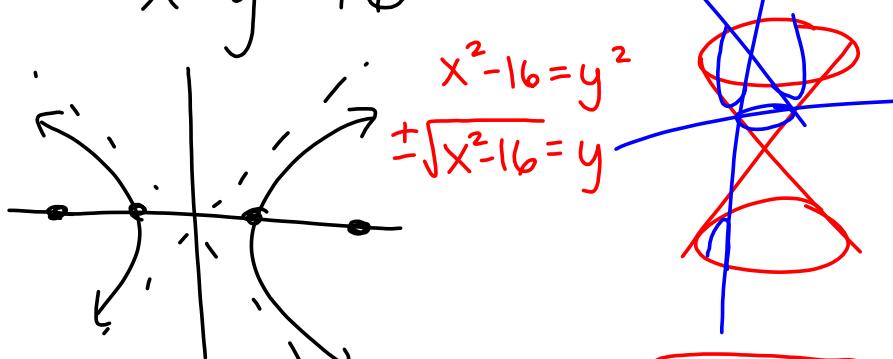
$$2xy + x^2y' + 2y \cdot y'x + y^2 \cdot 1 = 0$$

$$x^2y' + 2xyy' = -y^2 - 2xy$$

$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$y' = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

$$2. \quad x^2 - y^2 = 16$$



$$\frac{d}{dx} [x^2 - y^2] = \frac{d}{dx} [16]$$

$$2x - 2yy' = 0$$

$$2x = 2y y'$$

$$\cancel{\frac{2x}{2y}} = y'$$

$$y' = \frac{x}{y}$$

$$y' = \pm \frac{x}{\sqrt{x^2 - 16}}$$

$$8. \sqrt{xy} = x - 2y \quad (\sqrt{xy})' = x - 2y$$

$$\frac{d}{dx} [(\sqrt{xy})'] = \frac{d}{dx}[x - 2y]$$

$$\frac{1}{2}(xy)^{-\frac{1}{2}} \cdot (xy)' = 1 - 2y'$$

$$\frac{1}{2\sqrt{xy}} \cdot (xy' + 1 \cdot y) = 1 - 2y'$$

$$\frac{xy'}{2\sqrt{xy}} + \frac{y}{2\sqrt{xy}} = 1 - 2y'$$

$$\frac{xy'}{2\sqrt{xy}} + 2y' = 1 - \frac{y}{2\sqrt{xy}}$$

$$y' \left( \frac{x}{2\sqrt{xy}} + 2 \right) = 1 - \frac{y}{2\sqrt{xy}}$$

$$y' = \frac{1 - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} + 2}$$

$$10. \ 2\sin x \cos y = 1$$

$$\frac{d}{dx} [2\sin x \cos y] = \frac{d}{dx}[1]$$

$$(2\sin x)' \cos y + 2\sin x (\cos y)' = 0$$

$$2\cos x \cos y + 2\sin x (-\sin y)y' = 0$$

$$(-2\sin x \sin y) \cdot y' = -2\cos x \cos y$$

$$y' = \frac{-2\cos x \cos y}{-2\sin x \sin y} = \cot x \cot y$$

$$12. (\sin \pi x + \cos \pi y)^2 = 2$$

$$\frac{d}{dx} [(\sin \pi x + \cos \pi y)^2] = \frac{d}{dx} [2]$$

$$2(\sin \pi x + \cos \pi y) \cdot (\sin \pi x + \cos \pi y)' = 0$$

$$2(\sin \pi x + \cos \pi y)(\cos \pi x \cdot \pi + (-\sin \pi y) \cdot \pi y') = 0$$

$$(2\sin \pi x + 2\cos \pi y)(\pi \cos \pi x - \pi y' \sin \pi y) = 0$$

$$2\pi \sin \pi x \cos \pi x - 2\pi y' \sin \pi x \sin \pi y + 2\pi \cos \pi x \cos \pi y - 2\pi y' \sin \pi y = 0$$

$$2\pi \sin \pi x \cos \pi x + 2\pi \cos \pi x \cos \pi y = 2\pi y' \sin \pi x \sin \pi y + 2\pi y \sin \pi y \cos \pi y$$

$$\frac{2\pi \sin \pi x \cos \pi x + 2\pi \cos \pi x \cos \pi y}{2\pi \sin \pi x \sin \pi y + 2\pi \sin \pi y \cos \pi y} = y'$$

$$\frac{\cancel{2\pi \cos \pi x} (\sin \pi x + \cos \pi y)}{\cancel{2 + \sin \pi y} (\sin \pi x + \cos \pi y)} = y'$$

$$\boxed{\frac{\cos \pi x}{\sin \pi y} = y'}$$

$$16. x = \sec \frac{1}{y}$$

$$\frac{d}{dx}[x] = \frac{d}{dx} [\sec \frac{1}{y}]$$

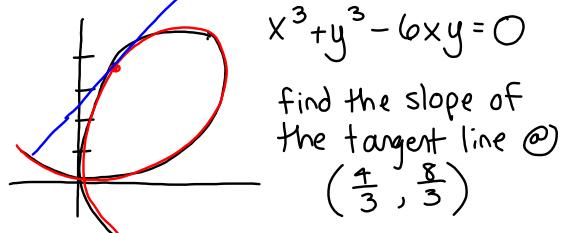
$$1 = \sec \frac{1}{y} \tan \frac{1}{y} \cdot (y^{-2})'$$

$$1 = \sec \frac{1}{y} \tan \frac{1}{y} \cdot (-y^{-2}) \cdot y'$$

$$y' = \frac{1}{\sec \frac{1}{y} \tan \frac{1}{y} (-y^{-2})}$$

$$y' = \boxed{-y^2 \cos \frac{1}{y} \cot \frac{1}{y}}$$

## 32. Folium of Descartes



$$\frac{d}{dx} [x^3 + y^3 - 6xy] = 0$$

$$3x^2 + 3y^2 \cdot y' - 6(1 \cdot y + x \cdot y') = 0$$

$$3x^2 + 3y^2 y' - 6y - 6xy' = 0$$

$$3y^2 y' - 6xy' = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{3(2y - x^2)}{3(y^2 - 2x)}$$

$$m = \frac{2(\frac{8}{3}) - (\frac{4}{3})^2}{(\frac{8}{3})^2 - 2(\frac{4}{3})} = \frac{\frac{16}{3} - \frac{16}{9}}{\frac{64}{9} - \frac{8}{3}} = \frac{\frac{48-16}{9}}{\frac{64-24}{9}} \\ = \frac{32}{40} = \boxed{\frac{4}{5}}$$

40. Find  $y''$  in terms of  $x$  &  $y$ .

$$y^2 = 4x$$

$$2yy' = 4$$

$$y' = \frac{4}{2y} = \frac{2}{y} = \boxed{2y^{-1}}$$

$$y'' = -2y^{-2} \cdot y'$$

$$= -2y^{-2} \cdot 2y^{-1} = -4y^{-3} = \boxed{-\frac{4}{y^3}}$$

Homework:

2.5 # 1-39odd; 43, 47 — *Implicit Differentiation*  
(2.6 #15-23odd; 25,27,35 )