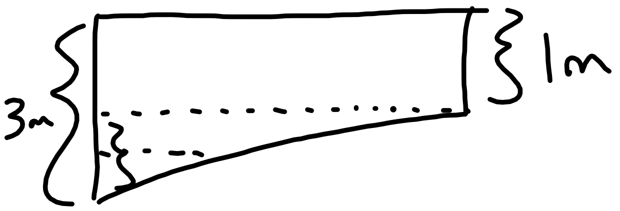
Homework questions?

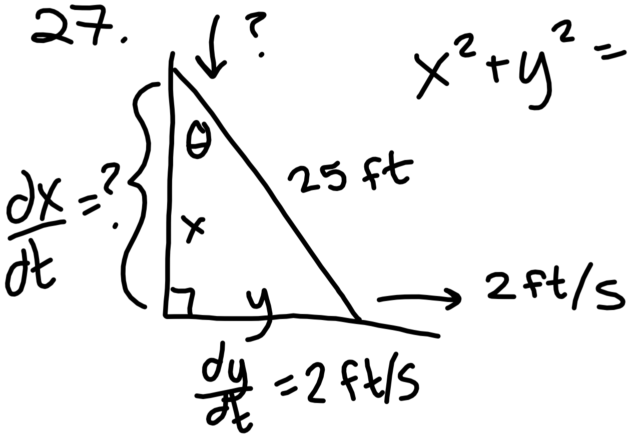
<u>Homework</u>:
2.5 # 1-39odd; 43, 47
2.6 #15-23odd; 25,27,35

25.



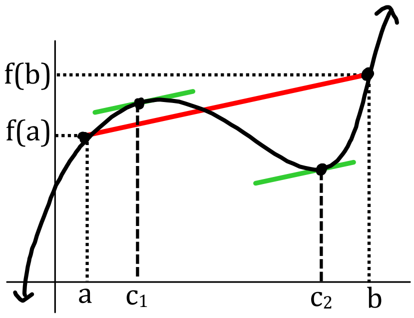
27.

$x^2+y^2=25^2$



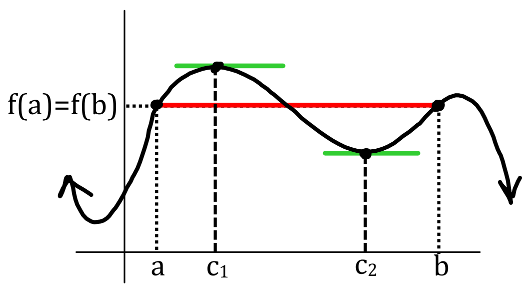
3.2 Rolle's Theorem & The Mean Value Theorem

The <u>Mean Value Thereom</u> (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



If f is cts. on $[a,b]$ & diff. on (a,b), $\exists\, c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

<u>Rolle's Theorem</u> is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



If f is cts. on $[a,b]$, diff on (a,b), and $f(a)=f(b)$, then $\exists\, c \in (a,b)$ s.t. $f'(c)=0$.

Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}, \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|, \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, ind all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4$, $[1,4]$

f cts on [1,4]? yes
f diff on (1,4)? yes
f(1) = f(4)? Yes

Rolle's Thm applies

$f(1) = 1^2 - 5(1) + 4 = 0$

$f(4) = 4^2 - 5(4) + 4 = 0$

$f'(x) = 2x - 5$

set $f'(c) = 0$

$2c - 5 = 0$

$2c = 5$

$\boxed{c = 5/2}$

Can the Mean Value Theorem be applied?
If so, ind all guaranteed values of c in (a,b).

$$34.\ f(x)=\frac{x+1}{x},\quad \left[\tfrac{1}{2},2\right]$$

<u>Steps to solve MVT problems</u>:
1. Is f continuous on [a,b]? yes
2. Is f differentiable on (a,b)? yes $\Rightarrow$ MVT applies
3. Find (f(b)-f(a))/(b-a)
4. Find f'(x)
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$$\frac{f(b)-f(a)}{b-a}=\frac{\frac{2+1}{2}-\frac{\frac{1}{2}+1}{1/2}}{2-1/2}=\frac{\frac{3}{2}-3}{3/2}=\frac{-3}{2}\cdot\frac{2}{3}=-1$$

$$f'(x)=\frac{x(1)-(x+1)\cdot 1}{x^2}=\frac{-1}{x^2}$$

$$-\frac{1}{c^2}=-1$$

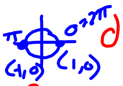
$$c=1$$

$$c=\pm 1$$

$$\boxed{c=1}$$

$$38.\quad f(x)=2\sin x+\sin 2x,\quad [0,\pi]$$

Is f cts on $[0,\pi]$? yes
diff on $(0,\pi)$? yes $\Rightarrow$ MVT applies.



$$\frac{f(b)-f(a)}{b-a}=\frac{(2\sin\pi+\sin 2\pi)-(2\sin 0+\sin 2(0))}{\pi-0}$$

$$=0$$

$$\cos 2x=2\cos^2x-1$$
$$=1-2\sin^2x$$
$$=\cos^2x-\sin^2x$$

$$f'(x)=2\cos x+2\cos 2x$$

$$2\cos x+2\underline{\cos 2x}=0$$

$$2\cos x+2(2\cos^2x-1)=0$$

$$\frac{4\cos^2x+2\cos x-2}{2}=\frac{0}{2}$$

$$2u^2+u-1$$
$$(2u-1)(u+1)$$

$$2\cos^2x+\cos x-1=0$$

$$(2\cos x-1)(\cos x+1)=0$$

$$2\cos x-1=0 \qquad \cos x+1=0$$

$$\cos x=1/2 \qquad \cos x=-1$$

$$\boxed{x=\frac{\pi}{3}} \qquad x=\pi$$

$$\pi\notin(0,\pi)$$

32. $f(x) = x(x^2 - x - 2)$ $[-1, 1]$

$= x^3 - x^2 - 2x$

f cts on $[-1,1]$? yes
f diff on $(-1,1)$? yes
$\Rightarrow$ MVT applies

$$\frac{f(b)-f(a)}{b-a} = \frac{1(1^2-1-2) - (-1)((-1)^2-(-1)-2)}{1-(-1)} = -1$$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\boxed{x = -1/3}$ $x = 1$

Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inlection Points and Concavity

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