Homework since Test #2 (Material for Test #3)

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval

due Mon

3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inlection Points and Concavity

<u>3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests</u>

What do *f′* and *f″* tell us about *f*?

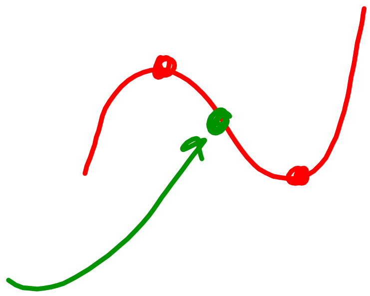
Recall that *f′* is the rate of change or slope of *f*,
f″ is the slope or rate of change of *f′*.

f'	f
+	$\nearrow$ increasing
−	$\searrow$ decreasing

f''	f'	f
+	$\nearrow$ increasing	concave up
−	$\searrow$ decreasing	concave down

f'(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undeined) are called <u>critical numbers</u>.

f''(x)=0 when f changes concavity.
The points where concavity changes are called <u>inlection points</u> .

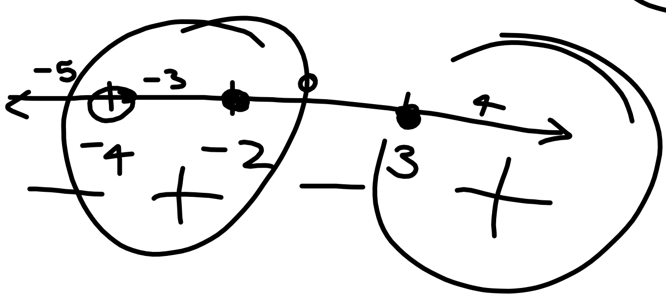


To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

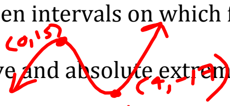
$$\frac{(x+2)(x-3)}{x+4} \geq 0$$

$-2 \quad 3 \quad -4$

$$\boxed{(-4,-2] \cup [3,\infty)}$$



- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.



3.3

$f(4) = 64 - 96 + 15$

16. $f(x) = x^3 - 6x^2 + 15$

$f(2) = 8 - 24 + 15$

$f'(x) = 3x^2 - 12x = 3x(x-4)$

Critical #'s $f'(x) = 0$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$\boxed{x = 0 \quad x = 4}$ critical #'s

$f'(-1)$, $f'(1)$, $f'(5)$

$+ \quad 0 \quad - \quad 4 \quad +$

f is increasing on $(-\infty, 0) \cup (4, \infty)$

f is decreasing on $(0, 4)$

f has a relative max @ $(0, 15)$ & a relative min @ $(4, -17)$

$f''(x) = 6x - 12$

inflection points: $f''(x) = 0$

$6x - 12 = 0$

$6x = 12$

$x = 2$

$(2, -1)$

$f''(0)$, $f''(3)$

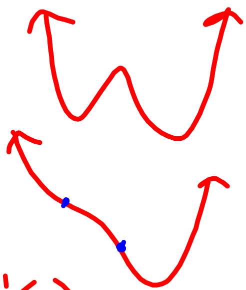
$- \quad 2 \quad +$

f is concave down on $(-\infty, 2)$

f is concave up on $(2, \infty)$

3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$



$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

critical #'s: 0 & 3

$f'(-1)$, $f'(1)$, $f'(4)$

$- \quad 0 \quad - \quad 3 \quad +$

f is decreasing on $(-\infty, 3)$

f is increasing on $(3, \infty)$

f has a relative minimum @ $(3, -27)$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

$f''(-1)$, $f''(1)$, $f''(3)$

$+ \quad 0 \quad - \quad 2 \quad +$

inflection points:

$(0, 0)$

$(2, -16)$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

f is concave down on $(0, 2)$

3.3

30. $f(x)=\dfrac{x+3}{x^2}$

$$f'(x)=\frac{x^2\cdot(1)-(x+3)\cdot 2x}{(x^2)^2}=\frac{-x^2-6x}{x^4}=\frac{-x(x+6)}{x^4}$$

$$=\frac{-x-6}{x^3}$$

$f'(-7)\ f'(-1)\ f'(1)$

$-\ \ -6\ \ +\ \ 0\ \ -$

$\boxed{\text{critical \#'s: } 0, -6}$

f is increasing on $(-6,0)$

f is decreasing on $(-\infty,-6)\cup(0,\infty)$

f has a relative min @ $(-6, -\frac{1}{12})$

$$f''(x)=\frac{x^3(-1)-(-x-6)\cdot 3x^2}{(x^3)^2}=\frac{2x^3+18x^2}{x^6}=\frac{x^2(2x+18)}{x^6}$$

$$=\frac{2x+18}{x^4}$$

inflection points: $(-9, \frac{-2}{27})$

$f''(-10)\ f''(-1)\ f''(1)$

$-\ \ -9\ \ +\ \ 0\ \ +$

f is concave down on $(-\infty,-9)$

f is concave up on $(-9,0)\cup(0,\infty)$

3.4 #20

$$f(x)=\frac{x+1}{\sqrt{x}}=\frac{x+1}{x^{1/2}}$$

critical #'s: 0, 1

$$f'(x)=\frac{x^{1/2}(1)-(x+1)\cdot\frac{1}{2}x^{-1/2}}{(x^{1/2})^2}$$

$$=\frac{x^{1/2}-\frac{1}{2}x^{1/2}-\frac{1}{2}x^{-1/2}}{x}$$

$$=\frac{\frac{1}{2}x^{1/2}-\frac{1}{2}x^{-1/2}}{x}\cdot\frac{x^{1/2}}{x^{1/2}}$$

$$=\frac{\frac{1}{2}x-\frac{1}{2}}{x^{3/2}}$$