

3.4 #20

$$f(x) = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}}$$

critical #'s: 0, 1

$$f'(x) = \frac{x^{1/2}(1) - (x+1) \cdot \frac{1}{2}x^{-1/2}}{(x^{1/2})^2}$$

$$= \frac{x^{1/2} - \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{x}$$

$$= \frac{\frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{x} \cdot \frac{x^{1/2}}{x^{1/2}}$$

$$f'(x) = \frac{\frac{1}{2}x - \frac{1}{2}}{x^{3/2}}$$

f is increasing on $(1, \infty)$

f is decreasing on $(0, 1)$

relative min @ $(1, 2)$

$$X^m X^n = X^{m+n}$$

$$f''(x) = \frac{x^{3/2}(\frac{1}{2}) - (\frac{1}{2}x - \frac{1}{2}) \cdot \frac{3}{2}x^{1/2}}{x^3} = \frac{\frac{1}{2}x^{3/2} - \frac{3}{4}x^{3/2} + \frac{3}{4}x^{1/2}}{x^3}$$

$$= \frac{-\frac{1}{4}x^{3/2} + \frac{3}{4}x^{1/2}}{x^3} \cdot \frac{x^{1/2}}{x^{1/2}} = \frac{-\frac{1}{4}x^2 + \frac{3}{4}x}{x^{5/2}} = \frac{-\frac{1}{4}x + \frac{3}{4}}{x^{5/2}}$$

$$-\frac{1}{4}x + \frac{3}{4} = 0$$

$x = 3$

inflection point @ $(3, \sqrt[4]{3})$

(undefined) $\begin{matrix} + \\ f''(1) \end{matrix}$ $\begin{matrix} - \\ f''(4) \end{matrix}$

$0 + 3 -$

f is concave up on $(0, 3)$
and concave down on $(3, \infty)$



3.4 #34 (Find all relative extrema; use the second derivative test where applicable)

$$\begin{aligned}
 g(x) &= -\frac{1}{8}(x+2)^2(x-4)^2 \\
 &\quad \left/ \begin{array}{l} g'(x) = -\frac{1}{8}(2(x+2)(x-4)^2 + (\frac{1}{8}(x+2)^2)(2(x-4))) \\ = -\frac{1}{8}(x+2)(x-4)(2(x-4) + (x+2)\cdot 2) \\ = -\frac{1}{8}(x^2 + 4x + 4)(x^2 - 8x + 16) \\ = -\frac{1}{8}(x^4 - 4x^3 - 12x^2 + 32x + 64x + 4x^2 - 32x + 64) \end{array} \right. \\
 &g(x) = -\frac{1}{8}x^4 + \frac{1}{2}x^3 + \frac{3}{2}x^2 - 4x - 8 \quad \left(\begin{array}{l} g''(x) > 0 \\ g''(x) < 0 \end{array} \right) \\
 &g'(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2 + 3x - 4 = -\frac{1}{2}(x^3 - 3x^2 - 6x + 8) \\
 &g''(x) = -\frac{3}{2}x^2 + 3x + 3 \quad \text{critical #'s: } (-2, 4) \\
 &= -\frac{3}{2}(x^2 - 2x - 2) \\
 &g''(-2) = - \max @(-2, 0) \\
 &g''(4) = - \max @ (4, 0) \\
 &g''(1) = + \min @ (1, -\frac{81}{8})
 \end{aligned}$$

1. Find the average rate of change of the volume of a sphere with respect to radius, as the radius of the sphere changes from 1 cm to 2 cm.

$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 \\
 \frac{\Delta V}{\Delta r} &= \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{2-1} \\
 &= \frac{\frac{32\pi}{3} - \frac{4\pi}{3}}{1} = \boxed{\frac{28\pi}{3} \text{ cm}^2}
 \end{aligned}$$

average
 rate of change
 = slope of
 secant line

2. Find the instantaneous rate of change of the volume of a sphere with respect to radius when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2 \Big|_{r=2} = 4\pi(2)^2 =$$

$$= 16\pi \text{ cm}^2$$

↓
slope of
tangent
line

3. If the radius of a sphere changes at a rate of 3cm per second, find the rate of change of the volume of the sphere with respect to time when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi(2)^2 \cdot 3$$

$$= 48\pi \text{ cm}^3/\text{s}$$

$$\frac{dr}{dt} = 3 \text{ cm/s}$$

$$\frac{dV}{dt} = ? \text{ when } r = 2 \text{ cm}$$

$$4. \quad x^3 + y^3 - 6xy = 0$$

$$3x^2 + 3y^2 \cdot y' - 6(1 \cdot y + x \cdot y') = 0$$

$$3x^2 + 3y^2 y' - 6y - 6xy' = 0$$

$$3y^2 y' - 6xy' = 6y - 3x^2$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \boxed{\frac{2y - x^2}{y^2 - 2x}}$$

$$5. \quad y = \sin(xy)$$

$$y' = \cos(xy) \cdot (1 \cdot y + x \cdot y')$$

$$y' = y \cos xy + xy' \cos xy$$

$$y' - xy' \cos xy = y \cos xy$$

$$y'(1 - x \cos xy) = y \cos xy$$

$$\boxed{y' = \frac{y \cos xy}{1 - x \cos xy}}$$

Homework since Test #2 (Material for Test #3)

- { 2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)
- { 3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inlection Points and Concavity

} HW
due
Friday

Take home quiz
due next wed.

Test #3 Next Wed!