

Determine if the Mean Value Theorem applies to the function on the given interval, and if so, find all values of  $c$  guaranteed by the theorem.

Review

$$f(x) = \sqrt{2-x}, \quad [-7, 2]$$

Domain:  $\{x | 2-x \geq 0\}$   
 $\{x | -x \geq -2\}$   
 $\{x | x \leq 2\}$   
 Cts on  $[-7, 2]$ ?  
 Diff on  $(-7, 2)$ ? } Yes  $\Rightarrow$  MVT applies

$f(x)$  is continuous at  $x=a$  if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

$f(x)$  is continuous on  $[a, b]$  if

$$\lim_{x \rightarrow c} f(x) = f(c) \text{ for all } c \in (a, b)$$

$$\& \lim_{x \rightarrow a^+} f(x) = f(a)$$

$$\& \lim_{x \rightarrow b^-} f(x) = f(b)$$

MVT  $\Rightarrow \exists c \in (a, b)$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(x) = \sqrt{2-x} = (2-x)^{1/2}$$

$$f'(x) = \frac{1}{2}(2-x)^{-1/2} \cdot (-1) = \frac{-1}{2\sqrt{2-x}}$$

$$\frac{f(2) - f(-7)}{2 - (-7)} = \frac{\sqrt{2-2} - \sqrt{2-(-7)}}{2 - (-7)} = \frac{-3}{9} = -\frac{1}{3}$$

$$\frac{-1}{2\sqrt{2-c}} = -\frac{1}{3}$$

$$-2\sqrt{2-c} = -3$$

$$\sqrt{2-c} = \frac{3}{2}$$

$$2-c = 9/4$$

$$c = 2 - \frac{9}{4}$$

$$c = -\frac{1}{4}$$

Find  $y'$  implicitly in terms of  $x$  and  $y$ .

Review

$$x^2 y + 3xy^3 = 5x^3 y^2$$

$$\underline{x^2 \cdot y'} + \underline{2x \cdot y} + \underline{3x \cdot 3y^2 y'} + \underline{3 \cdot y^3} = \underline{5x^3 \cdot 2y y'} + \underline{15x^2 \cdot y^2}$$

$$y'(x^2 + 9xy^2 - 10x^3y) = \frac{15x^2y^2 - 2xy - 3y^3}{x^2 + 9xy^2 - 10x^3y}$$

$$\underline{\cos x} + \underline{\sin y} = \underline{\tan(xy)}$$

$$-\sin x + y' \cos y = (\sec^2(xy)) \cdot (xy' + 1 \cdot y)$$

$$-\sin x + y' \cos y = \underline{xy' \sec^2(xy)} + y \sec^2(xy)$$

$$y'(\cos y - x \sec^2(xy)) = y \sec^2(xy) + \sin x$$

$$y' = \frac{y \sec^2(xy) + \sin x}{\cos y - x \sec^2(xy)}$$

Find the limit (if it exists).

Review

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x^2 - 1} \cdot \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2} =$$

$$\lim_{x \rightarrow 1} \frac{x+3-4}{(x^2-1)(\sqrt{x+3}+2)} = \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(x+1)(\sqrt{x+3}+2)}$$

$$= \frac{1}{(1+1)(\sqrt{1+3}+2)} = \boxed{\frac{1}{8}}$$

$$\lim_{x \rightarrow 2} f(x), \quad f(x) = \begin{cases} 10-x, & x \leq 2 \\ x^2 + 2x, & x > 2 \end{cases}$$

$$\left. \begin{array}{l} 10-2=8 \\ 2^2+2(2)=8 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = \boxed{8}$$

Slope = 2

Review

Find  $k$  such that the line  $y = 2x$  is tangent to the graph of the function  $f(x) = x^2 + kx$ .

$$\begin{cases} 2x = x^2 + kx \\ 2 = 2x + k \rightarrow \frac{2-k}{2} = x \end{cases} \quad f'(x) = 2x + k$$

substituting...

$$2\left(\frac{2-k}{2}\right) = \left(\frac{2-k}{2}\right)^2 + k\left(\frac{2-k}{2}\right)$$

$$\left(2-k = \frac{4-4k+k^2}{4} + \frac{2k-k^2}{2}\right) \cdot 4$$

$$8-4k = 4 - 4k + k^2 + 4k - 2k^2$$

$$8-4k = 4 - k^2$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)(k-2) = 0$$

$$\boxed{k=2}$$

Find the derivative of  $f$  with respect to  $x$ .

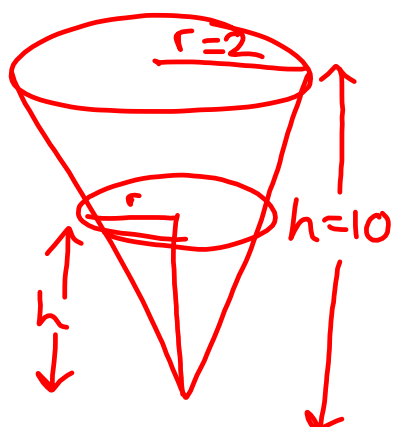
$$\sin^2 x = (\sin x)^2$$

$$f(x) = 5 \sin^2 \left( \sqrt{3 \csc(7x^2 - 2x)} \right)$$

$$= 5 \left( \sin \left[ (3 \csc(7x^2 - 2x))^{1/2} \right] \right)^2$$

$$\begin{aligned} f'(x) &= 10 \sin \left[ (3 \csc(7x^2 - 2x))^{1/2} \right] \cdot \\ &\quad \cdot \cos \left[ (3 \csc(7x^2 - 2x))^{1/2} \right] \cdot \\ &\quad \cdot \frac{1}{2} (3 \csc(7x^2 - 2x))^{-1/2} \cdot \\ &\quad \cdot (-3 \csc(7x^2 - 2x) \cot(7x^2 - 2x)) \cdot \\ &\quad \cdot (14x - 2) \end{aligned}$$

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$$\frac{dh}{dt} = \frac{1 \text{ in}}{5 \text{ min}}$$

$$\frac{dV}{dt} = ? \text{ when } h = 5 \text{ in}$$


$$V = \frac{1}{3} \pi r^2 h$$

Homework since Test #2 (Material for Test #3)


Submitted Tues, 01/14:

- 2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
- 2.6 # 15-23 odd - Related Rates
- 2.6 # 25, 27, 35 - Related Rates (more challenging problems)
- 3.1 # 17-31 odd - Absolute Extrema on an Interval

Due Fri, 01/17:

- 3.2 # 7-19 odd - Rolle's Theorem
  - 3.2 # 31-37 odd - Mean Value Theorem
  - 3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
  - 3.4 # 11-25 odd - Inflection Points and Concavity
- 

Due Wed, 01/22:

- Take-home quiz
  - (3.5 # 15-31 odd - limits at infinity)
- 

**Test #3 Wed, 01/22**

Upcoming:

- 7.7 # 11-35 odd - l'Hopital's rule
- 7.7 # 37-53 odd - l'Hopital's rule with logs
- 3.7 # 3, 5, 17, 23, 29 - optimization