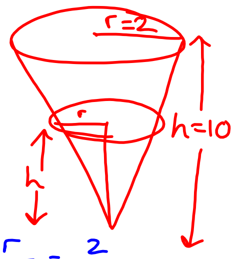
Turn in Homework #7:

- 3.2 # 7-19 odd - Rolle's Theorem
- 3.2 # 31-37 odd - Mean Value Theorem
- 3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
- 3.4 #11-25 odd - Inlection Points and Concavity

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$$\frac{r}{h} = \frac{2}{10}$$

$$r = \frac{h}{5}$$

$$\frac{dh}{dt} = \frac{-1 \text{ in}}{5 \text{ min}}$$

$$\frac{dV}{dt} = ? \text{ when } h = 5\text{in}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{5}\right)^2 \cdot h$$

$$V = \frac{1}{3} \cdot \frac{\pi}{25} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{25} h^2 \cdot \frac{dh}{dt}$$

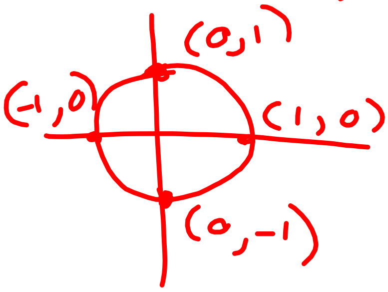
$$= \frac{\pi}{\cancel{25}} \cdot \cancel{(5)^2} \cdot \left(\frac{-1}{5}\right)$$

$$= \boxed{\frac{-\pi}{5} \frac{\text{in}^3}{\text{min}}}$$

$[0, 2\pi]$

$\cos x = 0$

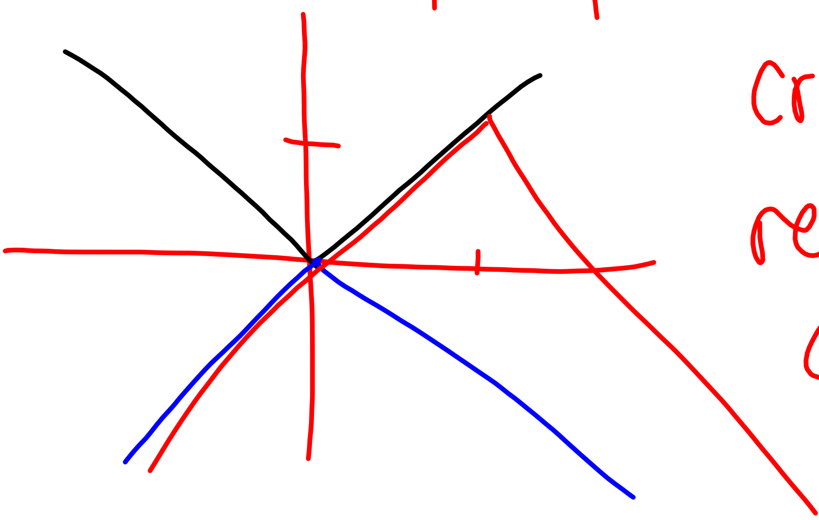
$x = \pi/2, 3\pi/2$



3.3

25. $f(x) = 5 - |x-5|$

$= -|x-5| + 5$



Critical #: 5

rel. max @ (5,5)

$f(x) = -3x \tan x$

a. Find $f'(x)$.

b. Find $f''(x)$.

$$f'(x) = -3\tan x + (-3x)\cdot \sec^2 x$$
$$= \boxed{-3\tan x - 3x\sec^2 x}$$

$$f''(x) = -3\sec^2 x + (-3)\sec^2 x + (-3x)\cdot\left[(\sec x)^2\right]'$$
$$= -3\sec^2 x - 3\sec^2 x - 3x(2\sec x \cdot \sec x \tan x)$$
$$= \boxed{-6\sec^2 x - 6x\sec^2 x \tan x}$$

2. $x^3 + y^2 = 10$

a. Find y' in terms of x and y.

b. Find y'' in terms of x and y.

$$3x^2 + 2y\cdot y' = 0$$
$$2yy' = -3x^2$$
$$\boxed{y' = \frac{-3x^2}{2y}}$$

$$y'' = \frac{(2y)(-6x) - (-3x^2)(2y')}{(2y)^2}$$
$$= \boxed{\frac{-12xy + 6x^2\left(\frac{-3x^2}{2y}\right)}{4y^2}}$$

3.5 Limits at Infinity

$\lim_{x \to \infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x) = \frac{5x^2 - 3x + 4}{2x^2 + 5x} \approx \frac{5x^2}{2x^2} = \frac{5}{2}$$

Horizontal asymptote
$y = 5/2$

$\Rightarrow \lim_{x \to \pm\infty} f(x) = 5/2$

$$f(x) = \frac{2x - 4}{3x^4} \approx \frac{2x}{3x^4} = \frac{2}{3x^3}$$

$\Rightarrow \lim_{x \to \pm\infty} f(x) = 0$

Horizontal asymptote
$y = 0$

$$f(x)=\frac{2x^7-4x^3-2}{5x^4+1} \approx \frac{2x^7}{5x^4}=\frac{2}{5}x^3$$

$$\lim_{x\to\infty} f(x)=\infty$$

$$\lim_{x\to-\infty} f(x)=-\infty$$

$$f(x)=\frac{2-7x^3+2x}{1+x} \approx \frac{-7x^3}{x}=-7x^2$$

$$\lim_{x\to\infty} f(x)=-\infty$$

$$\lim_{x\to-\infty} f(x)=-\infty$$

27. $$\lim_{x\to-\infty}\left(\frac{1}{2}x-\frac{4}{x^2}\right)$$

$$\frac{x}{2}\cdot\frac{x^2}{x^2}$$

$$=\lim_{x\to-\infty}\left(\frac{\frac{1}{2}x^3-4}{x^2}\right)$$

$$=\lim_{x\to-\infty}\left(\frac{1}{2}x\right)$$

$$=\boxed{-\infty}$$

$$=\lim_{x\to-\infty}\frac{1}{2}x-\lim_{x\to-\infty}\frac{4}{x^2}$$

$$=-\infty-0$$

$$=-\infty$$

26. $$\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$$

$$= \lim_{x\to -\infty} \frac{x}{\sqrt{x^2}}$$

$$= \lim_{x\to -\infty} \frac{x}{|x|}$$

$$= \lim_{x\to -\infty} \frac{x}{-x} = \lim_{x\to -\infty} (-1) = \boxed{-1}$$

$$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

28. $$\lim_{x\to -\infty} \frac{-3x+1}{\sqrt{x^2+1}}$$

$$= \lim_{x\to -\infty} \frac{-3x}{\sqrt{x^2}} = \lim_{x\to -\infty} \frac{-3x}{|x|} =$$

$$= \lim_{x\to -\infty} \frac{-3x}{-x} = \boxed{3}$$

$$\lim_{x\to\infty} \frac{5x-2}{\sqrt{9x^2+3}}$$

$$= \lim_{x\to\infty} \frac{5x}{\sqrt{9x^2}}$$

$$= \lim_{x\to\infty} \frac{5x}{3|x|} = \lim_{x\to\infty} \frac{5x}{3x} = \boxed{\frac{5}{3}}$$

30. $\lim_{x\to\infty} \frac{x-\cos x}{x}$

$$= \lim_{x\to\infty} \underbrace{\frac{x}{x}}_{=1} - \underbrace{\frac{\cos x}{x}}_{\to 0}$$

$\cos x \in [-1,1]$, $x \to \infty$

$$= \boxed{1}$$

32. $\lim_{x\to\infty} \cos\frac{1}{x}$

$= \cos\left[\lim_{x\to\infty}\frac{1}{x}\right]$

$= \cos 0$

$= \boxed{1}$

18. c.

$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1}$

$= \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}}$

$= \lim_{x\to\infty} \frac{5}{4}x$

$= \boxed{\infty}$

$\frac{5x}{4}$

$\frac{5}{4}\cdot\frac{x}{1} = \frac{5x}{4}$

Homework since Test #2 (Material for Test #3)

Submitted Tues, 01/14:
- 2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
- 2.6 # 15-23 odd - Related Rates
- 2.6 # 25, 27, 35 - Related Rates (more challenging problems)
- 3.1 # 17-31 odd - Absolute Extrema on an Interval

Due Fri, 01/17:
- 3.2 # 7-19 odd - Rolle's Theorem
- 3.2 # 31-37 odd - Mean Value Theorem
- 3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
- 3.4 #11-25 odd - Inlection Points and Concavity

Due Wed, 01/22:
- Take-home quiz
- (3.5 #15-31odd - limits at ininity) → Thurs. (not on Test)

Test #3 Wed, 01/22

Upcoming:
- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs
- 3.7 #3,5,17,23,29 - optimization