3.7 Optimization Problems

4. Find two positive numbers whose product is 192 and the sum of the irst plus three times the second is a minimum.

$$\frac{xy}{x} = \frac{192}{x}$$

$$S(x) = x + 3y$$

$$S(x) = x + 3 \cdot \frac{192}{x}$$

$$(x^{-1})' = -x^{-2}$$

$$\frac{1}{x}'' = \frac{-1}{x^2}$$

$$S'(x) = 1 + \frac{-3 \cdot 192}{x^2}$$

$$1 - \frac{3 \cdot 192}{x^2} = 0$$

$$x = \boxed{24}$$

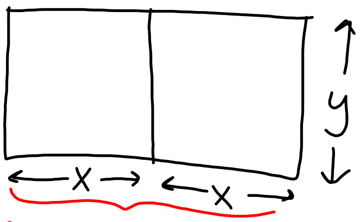
$$1 = \frac{3 \cdot 192}{x^2}$$

$$y = \frac{192}{24} = \boxed{8}$$

$$x^2 = 3 \cdot 192$$

$$x = \sqrt{3 \cdot 192} = \sqrt{576} = 24$$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the igure. What dimensions should be used so that the enclosed area will be a maximum?



$$4x + 3y = 200$$

$$3y = 200 - 4x$$

$$y = \frac{200}{3} - \frac{4x}{3}$$

$$A(x) = 2x \cdot y$$

$$A(x) = 2x\left(\frac{200}{3} - \frac{4x}{3}\right) = \frac{400}{3}x - \frac{8}{3}x^2$$

$$A'(x) = \frac{400}{3} - \frac{16}{3}x$$

$$\frac{400}{3} - \frac{16}{3}x = 0$$

$$\frac{400}{3} = \frac{16}{3}x$$

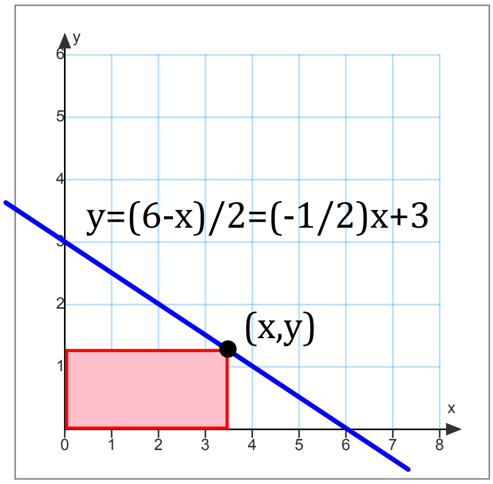
$$\frac{400}{16} = x$$

$$25 = x$$

$x = 25$ ft

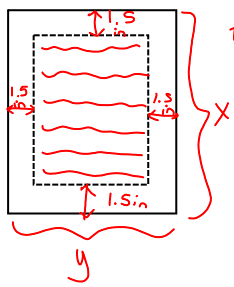
$y = \frac{100}{3}$ ft

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$$A = xy$$
$$A = x\left(\frac{6-x}{2}\right)$$
$$A = x\left(\frac{6}{2} - \frac{x}{2}\right)$$
$$A = 3x - \frac{1}{2}x^2$$
$$A'(x) = 3 - x$$
$$3 - x = 0 \Rightarrow \boxed{x = 3 \ \& \ y = 3/2}$$

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



$$A(x) = xy = x\left(\frac{36}{x-3} + 3\right)$$
$$36 = (x-3)(y-3)$$
$$\frac{36}{x-3} = y - 3$$
$$\frac{36}{x-3} + 3 = y$$
$$A(x) = \frac{36x}{x-3} + 3x$$
$$A'(x) = \frac{(x-3)\cdot 36 - 36x}{(x-3)^2} + 3$$
$$\frac{(x-3)\cdot 36 - 36x}{(x-3)^2} + 3 = 0$$
$$\frac{(x-3)\cdot 36 - 36x + 3(x-3)^2}{(x-3)^2} = 0$$
$$36x - 108 - 36x + 3(x^2 - 6x + 9) = 0$$
$$-108 + 3x^2 - 18x + 27 = 0$$
$$3x^2 - 18x - 81 = 0$$
$$3(x^2 - 6x - 27) = 0$$
$$3(x-9)(x+3) = 0$$
$$x = 9, \cancel{x = -3}$$

$$\boxed{x = 9\text{ in}}$$
$$y = \frac{36}{x-3} + 3$$
$$\boxed{y = 9\text{ in}}$$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^{∞}, 0^{0}, *and* $\infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form 0/0, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

7.7

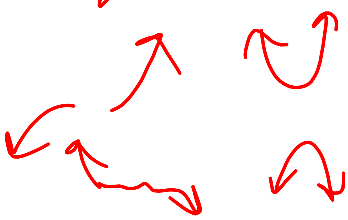
12. $\lim_{x \to -1} \frac{x^2 - x - 2}{x + 1} = \frac{0}{0}$

$= \lim_{x \to -1} \frac{2x - 1}{1}$

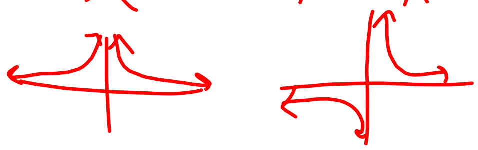
$= \boxed{-3}$

$$\lim_{x\to\infty} \frac{5x^2-3x+7}{29x^2-4} = \boxed{\frac{5}{29}}$$

$$\frac{c_1}{c_2} \; ; \; \frac{c_1 x^{odd}}{c_2} \qquad \frac{c_1}{c_2} x^{even} \qquad \frac{x^5}{x^2} \approx x^3$$



$$c \cdot \frac{1}{x^{even}} \; ; \; c \cdot \frac{1}{x^{odd}}$$



16. $$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{(e^x-1-x)'}{(x^3)'} = \lim_{x\to 0^+} \frac{e^x-1}{3x^2} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{(e^x-1)'}{(3x^2)'} = \lim_{x\to 0^+} \frac{e^x}{6x} = \frac{1}{0} \to \boxed{\infty}$$

$$\frac{1}{\frac{1}{10}}, \frac{1}{\frac{1}{100}}, \frac{1}{\frac{1}{1000}}$$

18. $\lim_{x\to 1} \frac{\ln x^2}{x^2-1} = \frac{0}{0}$

$\ln 1 = 0$

$\log_e 1 = 0$

$e^0 = 1$

$= \lim_{x\to 1} \frac{\frac{1}{x^2}\cdot \cancel{2x}}{\cancel{2x}}$

$= \lim_{x\to 1} \frac{1}{x^2} = \boxed{1}$

Homework:

→Limits at Ininity: 3.5 #15-31 odd
→L'Hopital's Rule: 7.7 #11-35 odd
L'Hopital's Rule with logs: 7.7 #37-53odd
→Optimization: 3.7 #3,5,17,23,29