

Our Exam is Wednesday, 12 Feb @ 9:00am

13. Find the limit.

a. $\lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{2x^2}$

b. $\lim_{x \rightarrow 3^-} f(x)$, where $f(x) = \begin{cases} \frac{x+2}{2}, & x \leq 3 \\ \frac{12-2x}{3}, & x > 3 \end{cases}$

c. $\lim_{x \rightarrow 3^+} \frac{x^2}{x^2 - 9}$

d. $\lim_{x \rightarrow \infty} \frac{3x^3 + 2}{9x^3 - 2x^2 + 7}$

e. $\lim_{x \rightarrow -\infty} \frac{3x - 5}{\sqrt{4x^2 + 2x - 1}}$

f. $\lim_{x \rightarrow 1} \frac{\ln x^2}{x^2 - 1}$

g. $\lim_{x \rightarrow \infty} \frac{e^{x/2}}{x^3}$

h. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

i. $\lim_{x \rightarrow \infty} (1 + x)^{1/x}$

$$\text{a. } \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{2x^2}$$

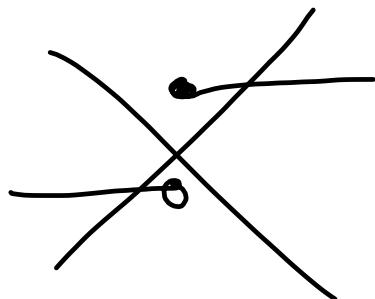
$$= \lim_{x \rightarrow 0} \frac{1}{2} \cdot \frac{\sin x}{x} \cdot \frac{1 - \cos x}{x}$$

$$= \frac{1}{2} \cdot 1 \cdot 0 = \boxed{0}$$

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

$$b. f(x) = \begin{cases} \frac{x+2}{2}, & x \leq 3 \\ \frac{2-2x}{3}, & x > 3 \end{cases}$$

$$\lim_{x \rightarrow 3^-} f(x) = \frac{3+2}{2} = \boxed{\frac{5}{2}}$$



$$c. \lim_{x \rightarrow 3^+} \frac{x^2}{x^2-9} = \lim_{x \rightarrow 3^+} \frac{x^2}{(x-3)(x+3)}$$

$$= \boxed{+\infty}$$

$3.001, \text{ e.g. ?}$
 $3^- 2.99, \text{ e.g. }$

$$d. \lim_{x \rightarrow \infty} \frac{3x^3 + 2}{9x^3 - 2x^2 + 7} = \lim_{x \rightarrow \infty} \frac{3x^3}{9x^3} = \boxed{\frac{1}{3}}$$

$$e. \lim_{x \rightarrow -\infty} \frac{3x-5}{\sqrt{4x^2+2x-1}}$$

$$= \lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{4x^2}} = \lim_{x \rightarrow -\infty} \frac{3x}{2|x|}$$

$$\sqrt{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$= \lim_{x \rightarrow -\infty} \frac{3x}{2(-x)}$$

$$= \boxed{-\frac{3}{2}}$$

$$f. \lim_{x \rightarrow 1} \frac{\ln x^2}{x^2 - 1}$$

$$= \lim_{x \rightarrow 1} \frac{\frac{1}{x^2} \cdot 2x}{2x} = \boxed{1}$$

$$g. \lim_{x \rightarrow \infty} \frac{e^{x/2}}{x^3} = \lim_{x \rightarrow \infty} \frac{e^{x/2} \cdot \frac{1}{2}}{3x^2}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{4} e^{x/2}}{6x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{8} e^{x/2}}{6} = \boxed{\infty}$$

$$h. \lim_{x \rightarrow \infty} \frac{\sin x}{x}$$

$$-1 \leq \sin x \leq 1$$

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\lim_{x \rightarrow \infty} -\frac{1}{x} \leq \lim_{x \rightarrow \infty} \frac{\sin x}{x} \leq \lim_{x \rightarrow \infty} \frac{1}{x}$$

$$0 \leq \lim_{x \rightarrow \infty} \frac{\sin x}{x} \leq 0$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{\sin x}{x} = \boxed{0}$$

(by the Squeeze Theorem)

$$i. \lim_{x \rightarrow \infty} (1+x)^{1/x}$$

$$y = \lim_{x \rightarrow \infty} (1+x)^{1/x}$$

$$\ln y = \lim_{x \rightarrow \infty} \ln(1+x)^{1/x}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln(1+x)$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\ln(1+x)}{x}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\frac{1}{1+x} \cdot 1}{x}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{1}{1+x}$$

$$\ln y = 0 \quad \boxed{y = 1}$$

5. Find the derivative of the function.

a. $f(x) = \sqrt{\frac{x}{x^3 - 2x}}$

c. $f(x) = 3x - 5 \cot(\pi x)^2$

b. $f(x) = \left(\frac{\sin x}{x^3 - 2x} \right)^3$

d. $f(x) = \ln(\tan^{-1}(2x))$

e. $f(x) = 5^{\csc x} \sqrt{x^3 - 7x}$

6. Find an equation of the tangent line to the graph of f at the indicated point.

$$f(x) = \sqrt{2x^2 - 7}, \quad (2, 1)$$

7. The length of a rectangle is given by $2(t^2 + 1)$ and its height is $\sqrt{t + 5}$ where t is time in seconds and dimensions are in centimeters.

a. Find the average rate of change of the area from time 4 to time 11.

b. Find the instantaneous rate of change of the area at time 4.

6. $f(x) = \sqrt{2x^2 - 7}, \quad (2, 1), \quad (x_1, y_1)$

$$f'(x) = \frac{1}{2} (2x^2 - 7)^{-1/2} \cdot 4x$$

$$= \frac{2x}{\sqrt{2x^2 - 7}}$$

$$m = f'(2) = \frac{2(2)}{\sqrt{2 \cdot 2^2 - 7}} = \frac{4}{1} = 4 = m$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 4(x - 2)$$

$$\boxed{y = 4x - 7}$$

$$7. A(t) = 2(t^2 + 1)\sqrt{t+5}$$

$$(a) = \frac{A(11) - A(4)}{11 - 4}$$

$$= \frac{2(11^2 + 1)\sqrt{11+5} - 2(4^2 + 1)\sqrt{4+5}}{11 - 4}$$

$$= \boxed{\quad}$$

$$(b) A'(t) =$$

$$A'(4) = \boxed{\quad}$$

14. Find the derivative of the function using the definition (limit of the difference quotient)

$$f(x) = x^3 - 12x$$

$$f'(x) = 3x^2 - 12$$

15. A rectangle is bounded by the x-axis and the semi-circle $y = \sqrt{25 - x^2}$ (see figure on p. 217 of textbook).

What length and width should the rectangle have so that its area is a maximum?

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - 12(x+h) - (x^3 - 12x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 12x - 12h - x^3 + 12x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2 - 12)}{h} = 3x^2 - 12$$

19. Find the critical numbers of f (if any), find the open intervals on which the function is increasing or decreasing, and locate all relative extrema.

$$f(x) = \frac{x^2 - 2x + 1}{x + 1}$$

$$\underline{g f' - g' f} = \underline{f' g - f g'}$$

20. Find the points of inflection and discuss the concavity of the graph of the function.

$$f(x) = 2x^4 - 8x + 3$$

$$\begin{aligned}
 19. f'(x) &= \frac{(x+1)(2x-2) - (x^2 - 2x + 1)(1)}{(x+1)^2} \\
 &= \frac{2x^2 - 2 - x^2 + 2x - 1}{(x+1)^2} = \frac{x^2 + 2x - 3}{(x+1)^2} \\
 &= \frac{(x+3)(x-1)}{(x+1)^2} \quad \text{critical #}'s -3, -1, 1 \\
 f'(-4) &\stackrel{+}{\cancel{f'(-3)}} \stackrel{+}{\cancel{f'(0)}} \stackrel{+}{\cancel{f'(2)}} \quad f \text{ is increasing on } (-\infty, -3) \cup (1, \infty) \\
 \uparrow &-3 \downarrow -1 \downarrow 1 \uparrow \quad f \text{ is decreasing on } (-3, -1) \cup (-1, 1) \\
 f &\text{ has a maximum } \circledcirc (-3, f(-3)) \\
 f &\text{ has a minimum } \circledcirc (1, f(1))
 \end{aligned}$$

$$f''(x) = 0$$

$$x = a, b, c$$

$$(a, f(a))$$

$$\begin{array}{c}
 \cancel{f''(\rightarrow)} \stackrel{+}{\cancel{f''(-)}} \stackrel{-}{\cancel{f''(+)}} \stackrel{+}{\cancel{f''(\sim)}} \\
 + a - b + c - \\
 \curvearrowleft \cap \curvearrowleft \curvearrowright
 \end{array}$$

$$\begin{array}{l}
 (b, f(b)) \\
 (c, f(c))
 \end{array}$$

17. Locate the absolute extrema on the closed interval.

$$f(x) = x^3 - 12x, [0, 4]$$

18. Determine whether the Mean Value Theorem can be applied to f on the closed interval, and if so, find all values of c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$$f(x) = \frac{x+1}{x}, \left[\frac{1}{2}, 2 \right]$$

$$[-4, 3]$$