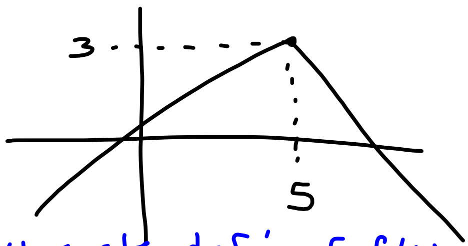


$f(x) = -|x-5| + 3$

f is continuous @ $x=5$, but not differentiable @ $x=5$

Alternate def'n of $f'(c)$

$$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

$$\lim_{x\to 5^-} \frac{-|x-5|+3-3}{x-5} = \lim_{x\to 5^-} \frac{-|x-5|}{x-5} = 1$$

$$\lim_{x\to 5^+} \frac{-|x-5|+3-3}{x-5} = \lim_{x\to 5^+} \frac{-|x-5|}{x-5} = -1$$

left- & right-hand limits are different

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$|-2| = -(-2) = 2$

2. Power Rule for $n \in \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Proof:

$$\frac{d}{dx}[c] = 0$$

Recall the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \frac{n!}{k!\,(n-k)!}a^{n-k}b^k + \cdots + b^n$$

$$[x^n]' = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + \ldots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \ldots + h^{n-1}\right)}{\cancel{h}}$$

$$= nx^{n-1}$$

Special case: $\frac{d}{dx}[x] = 1$

$$[x^1]' = 1 \cdot x^{1-1} = 1x^0 = 1$$

Examples:

$$\frac{d}{dx}[x^7] = 7x^{7-1} = \boxed{7x^6}$$

$$\frac{d}{dx}[\pi^3] = 0$$

$$\frac{d}{dx}[2e] = 0$$

$$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}\left[x^{1/2}\right] = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}\left[x^{-3}\right] = -3x^{-4} = \boxed{\frac{-3}{x^4}}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Examples:

$f(x) = 3x^2$

$f'(x) = 3 \cdot [x^2]' = 3(2x) = \boxed{6x}$

$f(x) = \frac{3}{x} = 3x^{-1}$

$f'(x) = -3x^{-2} = \boxed{\frac{-3}{x^2}}$

$g(x) = 2x^3 - x^2 + 3x$

$g'(x) = \boxed{6x^2 - 2x + 3}$

$y = 4x^{3/2} - 5x^4 + 2x^{\frac{1}{3}} - 7$

$y' = \boxed{6x^{1/2} - 20x^3 + \frac{2}{3}x^{-2/3}} = 6\sqrt{x} - 20x^3 + \frac{2}{3\sqrt[3]{x^2}}$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Proof that (sin x)'=cos x

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \; ; \; \lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$\frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$$

$$\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} =$$

$$= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$= \lim_{h\to 0}\left(\frac{\sin x\cos h-\sin x}{h}+\frac{\cos x\sin h}{h}\right)$$

$$= \lim_{h\to 0} \frac{-\sin x(1-\cos h)}{h} + \lim_{h\to 0}\frac{\cos x(\sin h)}{h}$$

$$= \lim_{h\to 0}(-\sin x)\cdot \lim_{h\to 0}\frac{1-\cos h}{h} + \lim_{h\to 0}\cos x \cdot \lim_{h\to 0}\frac{\sin h}{h}$$

$$= (-\sin x)\cdot 0 + \cos x \cdot 1$$

$$= \boxed{\cos x}$$

2.2

22. $y = 5+\sin x$

$y' = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3}+2\cos x = \frac{5}{8}x^{-3}+2\cos x$

$y' = \boxed{\frac{-15}{8}x^{-4} - 2\sin x}$

44. $h(x) = \frac{2x^3-3x+1}{x} = \frac{2x^3}{x}-\frac{3x}{x}+\frac{1}{x} = 2x^2-3+x^{-1}$

$h'(x) = \boxed{4x-x^{-2}}$

46. $y = 3x(6x-5x^2) = 18x^2-15x^3$

$y' = \boxed{36x-45x^2}$

52. $f(x) = \frac{2}{\sqrt[3]{x}}+3\cos x = 2x^{-1/3}+3\cos x$

$f'(x) = \boxed{-\frac{2}{3}x^{-4/3} - 3\sin x}$

2.2 cont.

$s(t)$ = position

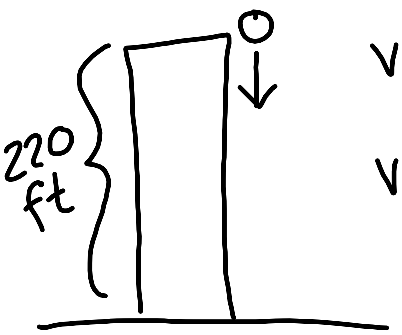
$v(t) = s'(t)$ = velocity

$a(t) = v'(t) = s''(t)$ = acceleration

average velocity: $\frac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92. initial velocity $v_0 = -22$ ft/s



$v(3) = ?$

$v(t) = ?$ after falling 108 ft

$g = -9.8\ m/s^2$

$= -32\ ft/s^2$

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

$s(t) = -16t^2 - 22t + 220$

$v(t) = s'(t) = -32t - 22$

$a(t) = v'(t) = -32$

Homework for Test #2 on Derivatives

HW #4 (due Fri 12/05)

- 2.1 #1-23 odd — Find the derivative by the limit process
  - #29-32 all — find the equation of the tangent line
  - #61-69 odd — Use the alternate form to find the derivative
  - #71-79 odd — Describe x-values where the function is differentiable (given graph)
- 2.2 #3-51 odd — Find the derivative using the basic derivative rules
  - #91-94 all; 101, 102 — use the derivative to solve rate of change word problems
- 2.3 #1-53 odd, 63-69 odd, 75-81 all, 83-91 odd, 109-115 all — Product and quotient rules
- 2.4 #7-33 odd — Chain rule

HW #5

- 2.4 #47-81 odd — Chain rule
- 5.1 #45-61, 71 — Logarithmic functions
- 5.4 #39-57 — Exponential functions
- 5.5 #41-55 — Log and exp functions with other bases
- 5.8 #41-59 — Inverse trig functions